

**Midterm II**

Name: \_\_\_\_\_

For the section that you *attend* please indicate:**Instructor:**(circle one)      Larget      Nordheim**Teaching assistant:**(circle one)      Cheng      Wilkinson      Zhang

## Instructions:

1. This exam is open book. You may use textbooks, notebooks, class notes, and a calculator (but not a laptop computer).
  2. Do all your work in the spaces provided. If you need additional space, use the back of the preceding page, indicating *clearly* that you have done so.
  3. To get full credit, you must show your work. Partial credit will be awarded.
  4. Note that some questions have multiple parts. For some questions, these parts are independent, and so you can work on part (b) separately from part (a).
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For graders' use:

Question	Possible Points	Score
1	18	
2	24	
3	14	
4	24	
5	20	
Total	100	

1. The underlined portions of the following statements are either TRUE or FALSE. Indicate which answer, TRUE or FALSE, you feel is more appropriate and give a justification for it. Note – no credit will be awarded without justification!
  - (a) Let  $T$  be distributed according to a t-distribution with 4 degrees of freedom and let  $Z$  be a standard normal.  $P(T \leq 1.533) < P(Z \leq 1.533)$
  - (b) Scientist X performs a paired two-sample t-test (on a properly designed experiment) on data from 10 pairs. X tests the null hypothesis that the mean difference is “0” versus the two-sided alternative and obtains a t-value of 1.72 which results in  $0.10 < p\text{-val} < 0.20$ . X argues that the original plan had been to use 20 pairs in this study but that last minute budget constraints allowed only for 10. X argues further that had 20 pairs been used and had the same values for the mean and standard deviation of the differences been obtained, the t-value would have been 2.43 ( $= 1.72 * \sqrt{2}$ ) resulting in a statement  $0.02 < p\text{-val} < 0.05$ . Thus, these are important results. You find X’s reasoning persuasive.

2. A dairy scientist conducted a study on the effect of two dietary supplements (formulated based on different mineral compositions) on anemia. Four cows were randomly selected to receive supplement A and four to receive supplement B. The data are composite mineral concentration (in  $\mu\text{g/d/Li}$ ) measured on blood samples from each cow taken after three months using the supplement.

Supplement A	5.7	7.1	6.6	5.0
Supplement B	5.8	4.7	3.8	5.3

- (a) Find a 90% confidence interval for the difference between the mean concentration in the blood for cows on supplement A and the mean concentration for cows on B.
  - (b) Based *only* on the confidence interval calculated in part(a), what conclusions can you reach about a test of the null hypothesis that the mean concentration in the blood for cows on A is exactly 0.3 larger than the mean concentration for cows on B (versus the two-sided alternative). Explain.
3. A study was undertaken to compare the heights of six different corn varieties five weeks after planting. It is assumed that for each variety, the heights are normally distributed with a variety-specific mean but a common variance. Also, all observations are independent. Given below is a table of sample sizes, means, and standard deviations for the recorded data. (The units are centimeters.)

Variety	1	2	3	4	5	6
n	10	10	7	6	9	10
$\bar{y}$	57.6	83.2	64.7	50.3	77.4	75.9
s	12.4	14.8	8.5	17.3	10.3	11.0

Find the pooled variance that results from these data. How many degrees of freedom are associated with this variance?

4. A study was undertaken to assess the prevalence of a rare bacteria in feline stomachs.
  - (a) It is hypothesized that the rate of occurrence of this bacteria in Siberian tigers (in US zoos) is 0.05 or less. A random sample of 15 Siberian tigers was selected and the bacteria was present in the stomachs of exactly two tigers. Perform a test of the hypothesis and interpret the result.
  - (b) A follow-up study is planned for domestic cats (in the US). For cats it is hypothesized that the rate of occurrence of this bacteria is 0.15 or less. In this study, a random sample of 60 cats will be selected and the proportion of them with this bacteria present in the stomach will be determined. It is decided that the null hypothesis will be rejected if the observed proportion (of cats with this bacteria in the stomach) is at least 0.25. Find the probability of Type I error ( $\alpha$ ) for this test.

5. A scientist is planning a study to compare the success rates of two different methods (A and B) of gene insertion into soy bean seeds. Based on prior work the scientist anticipates that the success rate for A will be around 0.4 and the success rate for B will be around 0.6. The scientist is planning on performing a total of 100 independent gene insertions but is considering various factors in selecting how many should be performed with each method.
- (a) Suppose that the goal is to minimize the width of a 95% confidence interval for the difference between the proportion of successes for A and the proportion of successes for B. Two choices are initially under consideration. (1) Perform 90 insertions with A (and 10 with B); or (2) Perform 70 insertions with A (and 30 with B). Which of these choices results in the narrower confidence interval? (Make sure that your conclusion is adequately supported.)
- (b) Based on your work on part (a) of this problem, make a conjecture for the allocation of trials to the two methods (with 100 total trials) that will result in the narrowest confidence interval. Explain your reasoning.