

**Tests to compare one sample of continuous values with a claim** about the true mean  $\mu_0$ . Null hypothesis  $H_0: \mu = \mu_0$ . Common assumption: The sample is a random sample, i.e. independence of observations.

Test	Variance $\sigma^2$	Distribution of $Y_i$	test statistic	null distribution
Z-test	known	$\mathcal{N}(\mu, \sigma^2)$	$Z = \frac{\bar{Y} - \mu}{\sigma/\sqrt{n}}$	$\mathcal{N}(0, 1)$
T-test	unknown	$\mathcal{N}(\mu, \sigma^2)$	$T = \frac{\bar{Y} - \mu}{S/\sqrt{n}}$	$T_{n-1}$
T-test	unknown	Any, provided $n$ is large	$T = \frac{\bar{Y} - \mu}{S/\sqrt{n}}$	$T_{n-1} \approx \mathcal{N}(0, 1)$

**Tests to compare one sample of binary values with a claim** about the true proportion  $\pi$  of success. Null hypothesis  $H_0: \pi = \pi_0$ . Common assumptions, required for the number of successes  $Y$  to be  $\mathcal{B}(n, \pi)$  distributed: random sampling (independence of observations) and a constant probability of success  $\pi$  across trials.

Test	sample size	test statistic	null distribution	the test is
Z-test	$n\pi$ and $n(1 - \pi) \geq 5$	$Z = \frac{Y - n\pi}{\sqrt{n\pi(1 - \pi)}}$	$\mathcal{N}(0, 1)$	approximate
Chi-square test	$n\pi$ and $n(1 - \pi) \geq 5$	$X^2$	$\chi^2$ on 1 df	approximate
Binomial test	Any. typically used with small $n\pi$ or $n(1 - \pi)$	$Y$	$\mathcal{B}(n, \pi)$	exact

**Tests to compare one sample of categorical values with a claim** about the true proportions  $p_j$  of categories. Null hypothesis  $H_0: p_1 = p_1^0$  and  $p_2 = p_2^0, \dots$  and  $p_r = p_r^0$ . Assumption: random sampling and constant probabilities of category membership  $p_j$  across trials.

Test	sample size	test statistic	null distribution	the test is
Chi-square goodness-of-fit	Expected counts $> 5$ in 80% cells and $> 1$ in all cells	$X^2$	$\chi^2$ on $(r - 1)$ df	approximate

**Tests to compare the means of 2 paired samples**, continuous values. Null hypothesis  $H_0: \mu_1 = \mu_2$ . Common assumption: each sample is a random sample of size  $n = n_1 = n_2$ , and each observation in one sample is paired to another observation in the other sample.

Test	variance of $D$	Distribution of $D$	test statistic	null distribution
Paired z-test	known	$\mathcal{N}(\mu_d, \sigma_d^2)$	$Z = \frac{\bar{d} - \mu_d}{\sigma_d/\sqrt{n}}$	$\mathcal{N}(0, 1)$
Paired t-test	unknown	$\mathcal{N}(\mu_d, \sigma_d^2)$ or large $n$	$T = \frac{\bar{d} - \mu_d}{S_d/\sqrt{n}}$	$T_{n-1}$
Signed rank test	any	any	(not covered)	

**Tests to compare the means of 2 independent samples**, continuous values. Null hypothesis  $H_0: \mu_1 = \mu_2$ . Common assumption: random sampling (independence) within samples, and independence across samples.

Test	variances $\sigma_1^2, \sigma_2^2$	Distribution of $Y_{i,j}$	test statistic	null distribution
Z-test	known	$\mathcal{N}(\mu_1, \sigma_1^2),$ $\mathcal{N}(\mu_2, \sigma_2^2)$	$Z = \frac{(\bar{Y}_1 - \bar{Y}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$	$\mathcal{N}(0, 1)$
T-test	unknown, $\sigma_1 = \sigma_2$	$\mathcal{N}$ or: large $n_1$ and large $n_2$	$T = \frac{(\bar{Y}_1 - \bar{Y}_2) - (\mu_1 - \mu_2)}{S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$	$T_{n_1+n_2-2}$
T-test	unknown	$\mathcal{N}$ or: large $n_1$ and large $n_2$	$T = \frac{(\bar{Y}_1 - \bar{Y}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$	$T_{\text{adf}}$
Mann-Whitney	similar	any	if $n_1 \leq n_2$ : min of $RS_1$ and of $n_1(n_1 + n_2 + 1) - RS_1$	tabulated

**Test to compare the means of k independent samples**, continuous values.

Null hypothesis  $H_0: \mu_1 = \mu_2 = \dots = \mu_k$ . Common assumption: Random sampling (independence) within samples, and independence across samples.

Test	variances $\sigma_1^2, \sigma_2^2$	Distribution of $Y_{i,j}$	test statistic	null distribution
ANOVA F-test	equal	$\mathcal{N}$ or: large $n_i$ 's	$F = \frac{MSTreatment}{MSError}$	$F_{k-1, N-k}$
Kruskall-Wallis	similar	any	based on ranks	(not covered)

**Test to compare the variances of k independent samples**, continuous values.

Null hypothesis  $H_0: \sigma_1^2 = \sigma_2^2 = \dots = \sigma_k^2$ . Assumption: random sampling (independence) within samples, and independence across samples.

Levene's test: apply the ANOVA F-test on transformed data (roughly: on the absolute deviations). If  $k = 2$ , the ANOVA F-test is equivalent to the independent t-test that assumes equal variances.

**Test of association: to compare proportions across k independent samples**, categorical observations.

Let  $r =$  number of categories. Null hypothesis  $H_0: p_1$  is the same in all groups, and  $p_2$  is the same in all groups, and  $\dots$ , and  $p_r$  is the same in all groups. Equivalently,  $H_0$  means that the **group membership is independent of the category membership**, i.e. no association. Assumption: Random sampling.

Test	sample size	test statistic	null distribution
Chi-square test of association	Expected counts $> 5$ in 80% of cells and $> 1$ in all cells	$X^2$	$\chi^2$ on $(k - 1) * (r - 1)$ df