1. Mark whether each item is True or False. If True, you may add a brief clarification, but a correct answer without explanation will receive full credit (The clarification might be worth partial credit if the correct response was False. If False, briefly explain why.

   (a) A t distribution with 5 degrees of freedom has the same shape as a standard normal distribution, but it has more density near zero than a normal does. □ True □ False

   (b) Two events A and B in the same sample space S are independent if $P(A|B) = P(B)$. □ True □ False

   (c) Given two independent random variables $X$ and $Y$ with $\text{var}(X) = 10$ and $\text{var}(Y) = 12$, we have that $\text{var}(X - Y) = 22$. □ True □ False

   (d) Consider two sets $A = \{1, 2, 7, 10\}$ and $B = \{3, 7, 11\}$. The intersection of these sets is $A \cap B = \{7, 10\}$. □ True □ False

   (e) The p-value is always calculated assuming that the null hypothesis is true. □ True □ False

   (f) A small p-value means that the null hypothesis has a small probability of being true, and leads to rejecting this null hypothesis. □ True □ False
2. The following problems are not related.

(a) Chewing lice (Mallophaga) feed on the feathers of some birds. An scientist collected data on 160 birds. He counted the number of lice present on one randomly sampled feather of each bird. He found \( \bar{x} = 20 \) lice/feather on average, with standard deviation \( s = 5 \) lice/feather. The distribution was skewed right. Draw a possible histogram for these data, representing the features specified above. The histogram must include horizontal and vertical axes with scales and labels.

(b) The nutshells of the shagbark hickory (Carya ovata) are very hard and sometimes must be cracked with a vice. Five randomly chosen nuts were cracked in a special vice which was able to determine the amount of pressure required to crack the shell. The mean cracking pressure required was \( \bar{x} = 200 \) pounds per square inch (psi). The first 4 data points deviated from the mean value by -5, 7, -1, and -6 psi. Calculate the standard deviation for this sample.

3. Over its lifetime, semen from bull A has resulted in conception in 30% of artificially inseminated Holstein dairy cattle. For bull B, this rate is 45%. A breeder has 20 vials of semen, 7 from bull A and 13 from bull B, but he does not know which vials belong to each bull. If he chooses one vial at random and artificially inseminates one cow, what is the probability that the cow conceives?
4. In the wild, the average Humboldt penguin (*Spheniscus humboldti*) has a bill length of 6.4 cm. A researcher is curious whether the carefully regulated diet of captive Humboldts affects their bill length, either positively or negatively. Note that all zoos use the same standard diet for penguins. She samples 19 penguins from zoos all over the world (one penguin from each zoo) and finds that the average bill length of the captive penguins is 6.1 cm, and the sample standard deviation of the bill lengths is 1.2 cm. Observed bill lengths seemed approximately normally distributed.

(a) Define null and alternative hypotheses that would be useful in answering the researcher’s question.

(b) Perform a test of the hypotheses that you defined in (a), and compute a p-value. Does the evidence seem to support the null hypothesis, or not? Explain.

5. Oriental bittersweet *Celastrus Orbiculatus* is a non-native invasive plant in the US. A large area was sown with 120 seeds of oriental bittersweet. This area was located in a stand of tulip poplars. Seeds were placed away from each other, so that germination may be assumed to be independent across seeds. Out of these 120 seeds, exactly 58 germinated. The germination rate of oriental bittersweet seeds is known to be 0.32 when sown in stands of oak trees. Determine if the germination rate in tulip poplar stands differs (or not) from 0.32. Show your different steps; state your hypotheses and conclusions clearly.
6. In a different experiment, 40 plots were surveyed for the presence of oriental bittersweet. Of these 40 plots, 10 plots were adjacent to the road and in historically agricultural (HAg) areas, 10 plots were 50 meters away from the road and in HAg areas, 10 plots were adjacent to the road and in reference (Ref) areas (areas never used for agriculture), and 10 plots were 50 meters away from the road in Ref areas. These plots can be assumed to be independent. Assume for this problem that the distance from the road does not affect the probability that oriental bittersweet is present. Also assume that the probability of oriental bittersweet presence is known to be \( p_a = 0.72 \) in HAg areas.

(a) What is the distribution of \( X_{a,50} \), the number of plots where oriental bittersweet is present, out of the 10 plots located 50 meters away from the road in HAg areas? Also describe the shape of this distribution (bell-shaped, bimodal, skewed left or right?).

(b) Find the probability that oriental bittersweet is found in exactly 6 of the 10 plots that are located near the road in HAg areas.

(c) Find the probability that oriental bittersweet is found in exactly 6 of the 10 HAg plots located near the road or exactly 6 of the HAg plots 50 meters away from the road.

(d) Let \( p_r \) be the probability of oriental bittersweet presence in Ref areas. The scientist has a suspicion that bittersweet invasibility is lower in ‘untouched’ reference areas than in previously agricultural areas, so he would like to know how \( p_r \) compares to 0.72. Out of the 10 plots that are located near the road in reference areas, exactly 1 was found to have oriental bittersweet. Based on these data, perform a test to determine if \( p_r = 0.72 \).