1. Twenty tulip petals were randomly chosen from a large tulip field in the Netherlands. The field was part of an experiment in breeding tulips to increase their redness. Each petal then underwent spectrophotographic analysis to determine its redness on a standardized scale of 0 to 100 redness units (where 0 = no red at all, and 100 = nothing but red, with values only on whole numbers). The original data were not available, but there exists the following summary information: Let $x_i$ denote the redness value for petal $i$. then $\sum_{i=1}^{20} x_i = 1044$, and $\sum_{i=1}^{20} x_i^2 = 56172$; and the following histogram (note that the ranges above each bar indicate the lower and upper endpoints of that bin):

(a) Find the sample mean petal redness.

(b) Find the sample standard deviation of petal redness.

(c) How many petals in the sample had a redness of between 40 and 49 units, inclusive?
2. Mark whether each item is True or False. If the item is False, briefly explain why.

(a) I have two events $A$ and $B$, both on the sample space $S$. $A \cup B = S$, so $A$ and $B$ must be mutually exclusive.

(b) Let $X \sim Bin(n, p)$. Additionally, let $Y = 2X$. It follows that $Y \sim Bin(2n, p)$.

(c) In a test of $H_0 : \mu = 10$ vs. $H_A : \mu \neq 10$, using $\bar{X}$ as an estimator of $\mu$, suppose we find $\bar{x} = 15$ and $p$-value $= 0.0001$. Even though this is a two-sided hypothesis test, there is more evidence that $\mu$ is greater than 10 than there is that $\mu$ is less than 10.

(d) Suppose we take a sample from a large population, and all the data values we get in our sample are negative. In this case, the sample variance as computed on our sample will be negative too.

(e) Suppose we take a sample of size $n = 20$ from a large population. A histogram with 25 equally sized bins will likely result in an appropriate graphical summary of this information, if the general shape of the distribution is of interest.

(f) We said in class that the normal approximation to the Binomial is appropriate when $n \times p > 5$ and $n \times (1 - p) > 5$, where $n$ is the number of trials and $p$ is the probability of success on each trial. The reason that $n$ is required to be ‘less large’ here than in a typical application of the Central Limit Theorem (the book states $n > 30$), is because the normal approximation to the binomial is not based on the CLT.
3. I have 10 snails in an aquarium, 5 of each of two species, call them species A and species B. The species appear identical to the naked eye. I have numbered the shells of these snails 1 through 10 to help me in taking a random sample. I have two sampling options:

Option 1: Sample the first snail, and set it aside. Then pick the second snail. This is sampling ‘without replacement’.

Option 2: Sample the first snail, put it back in the aquarium, and then pick the second snail. This is sampling ‘with replacement’.

(a) I will sample according to Option 1. What is the probability that I get two snails of species A?

(b) I will sample according to Option 2. What is the probability that I get two snails of species A?

(c) I will sample according to Option 1. Consider the events $A_1 =$ first snail chosen is of species A, and $A_2 =$ second snail chosen is of species A. Are these events independent? Explain why by showing that the mathematical definition of independence either holds or does not hold in this case.
4. (a) For the following sampling scenarios, state whether the binomial distribution would describe the probability distribution of possible outcomes. If you answer no, briefly explain why.

i. The number of red flowers in a square meter in a field. □ Yes □ No

ii. The number of red-eyed flies (which is the normal condition) among 150 *Drosophila* individuals drawn at random from a large population. □ Yes □ No

iii. The total number of red-eyed flies in five *Drosophila* families, each family consisting of 30 genetically related individuals, with the families chosen at random from a large population. □ Yes □ No

(b) Murphy’s law states that pieces of toast that are buttered on only one side have a higher chance of landing butter-side down when dropped. Let \( p \) be the probability of landing butter-side down.

i. State a null and alternative hypothesis in terms of \( p \) that could be used to test Murphy’s law. Justify your choice of alternative.

ii. An experiment is finally conducted: \( n \) independent pieces of toast are buttered on one side and then dropped. Assuming \( p = .65 \) and the experimenter uses \( n = 11 \), calculate the probability that exactly 5 slices of toast land butter-side down.

iii. Suppose \( n = 98 \). Assuming \( p = .5 \), calculate the mean and standard deviation for the number of slices that land butter-side down in this experiment.

iv. Still assuming \( n = 98 \) and \( p = .5 \), calculate the probability that at least 61 slices land butter-side down.
5. In European earwigs, the males sometimes have very long pincers protruding from the end of their abdomen.

(a) The graph below represents the distribution of pincer lengths in a particular population of European earwigs (in mm). The distribution is approximately normal. Each of the two gray areas represents 2.5% of this population, i.e. 2.5% of the area under the curve. Estimate the following quantities from the graph:

i. The mean

ii. The standard deviation

(b) Based on your previous estimates, complete the following sentence: “In the population described in (a), about 70% of male earwigs have pincers of length at most \( \text{mm} \).”

Show your work below.

(c) In a different population, \( n \) male earwigs have been sampled, measured, and the average of their \( n \) pincer lengths has been calculated. This procedure has been repeated many times and we have plotted below the frequency distribution of sample means based on three sample sizes: \( n = 1 \), \( n = 2 \) and \( n = 8 \). Identify which frequency distribution corresponds to each sample size.

Explain the basis for your decisions.
6. One researcher studied ruby-throated hummingbirds and concluded that adult male ruby-throated hummingbirds have a weight that is normally distributed with mean 3.2g and standard deviation 0.4g. Another researcher would like to know if the weight of females differs from that of males, in order to decide if males and females should be combined in a future analysis. She already knows that females tend to be bigger than males in some related hummingbird species. That researcher assumes that the weight of adult females is also normally distributed with standard deviation 0.4g and wants to test the hypothesis that the mean weight of adult females is the same as that of males: 3.2g.

Two of her graduate students (Joe and Jack) carried out their own separate data collections and each correctly performed a z-test on their data. They obtained two different results: Joe obtained a p-value of 0.65 and Jack obtained a p-value of 0.018.

(a) How is this possible? Explain at least two potential reasons.

(b) Which of Joe or Jack obtained the larger z-score (in absolute value)? Explain the reasoning behind your answer.

(c) The researcher combines Jack’s and Joe’s data, which makes a total of 30 female hummingbirds with a sample mean weight of 3.307g. State null and alternative hypotheses, and justify your choices. Next, compute the p-value for your test. What do you conclude?