1. Fennec Foxes (Vulpes zerda) have very long ears, which primarily function to dissipate heat in their very warm Saharan habitat. The distribution of the distance from the chin to the tip of the ears, $X$, is closely approximated by a normal rv, with mean 25cm, and variance 4 cm$^2$, that is, $X \sim \mathcal{N}(25, 4)$.

A local zoo is building a fennec enclosure, and they want to ensure that the fennec will be able to hide completely behind a small wall if it wants to, but they have not yet captured the animal. How tall should the wall be constructed so that 98% of all fennecs captured would be able to hide behind such a wall? That is, find $x$ such that $\Pr\{X < x\} = 0.98$.

2. The Magnificent Spider (Ordgarius magnificus) of Australia does not spin a web, but instead catches prey by spinning a short line of silk with a sticky glob on the end. The spider swings the line at approaching prey, and, if she scores a hit, the prey can then be reeled in and eaten. A researcher studying the Magnificent Spider has taken 15 very accurate measurements on the lengths of these silk lines (in cm). Here are the data:
7.808, 8.645, 8.625, 8.609, 8.581, 8.571, 8.435, 7.428, 8.366, 8.362, 8.193, 8.183, 8.132, 8.012, 9.015

(a) The researcher would like to present a graphical display of these data. In your opinion, does this data lend itself better to a stem and leaf plot, or a histogram? Justify your answer.

(b) Find the 0.18 quantile \( x_{0.18} \) for these data.

(c) Find the sample variance of this data set. You may use the facts that \( \sum x_i = 124.965 \) cm and \( \sum x_i^2 = 1043.218 \) cm\(^2\).

(d) The researcher originally had 16 observations, but her toddler son escaped from his playpen and erased the 16th observation. By coincidence, she knows that the value that was erased was exactly equal to the sample mean of the original 16 observations. Without calculation but with a precise and concise justification, determine whether

   i. the sample mean of the original 16 observations is \(\Box\) larger than, \(\Box\) smaller than, or \(\Box\) equal to the sample mean of the available 15 observations,

   ii. the sample variance of the original 16 observations is \(\Box\) larger than, \(\Box\) smaller than, or \(\Box\) equal to the sample variance of the available 15 observations.
3. In a population, 92% of all baby girls get immunized against Rubella early in their childhood (and 8% do not) because it is important for pregnant women to have been immunized prior to pregnancy. On the other hand, 40% of all baby boys get immunized against Rubella in their early childhood and 60% do not. The sex ratio is 1:1 in this population, meaning that there are equally many females and males.

(a) Suppose that a single child is sampled at random from the population. Let $B$ be the event “the child is a boy” and let $I$ be the event “the child is immunized against Rubella”.

i. Calculate the probability $P(B)$ that the child is a boy and the probability $P(I)$ that the child is immunized.

ii. Are these two events $B$ and $I$ independent?

(b) Suppose now that two (2) children from the same sex are sampled at random from the population. Specifically, a first child is sampled at random without sex restriction, and then another child of the same sex as the first child is chosen at random from the population. Let $Y$ be the number of children immunized against Rubella among the 2 selected children. Then $P\{Y = 0\} = 0.1832$ and $P\{Y = 1\} = 0.3136$.

i. Determine the probability $P\{Y = 2\}$ that both children are immunized.

ii. Calculate the expectation $E(Y)$ and variance $\text{var}(Y)$ of $Y$.

iii. Does $Y$ have a binomial distribution? □ Yes □ No. Why, or why not?
4. Sex determination in sea turtles is temperature driven, with high temperatures producing females and low temperatures producing males. In a given species of sea turtles, laboratory experiments showed that eggs have a 25% chance to turn out females and 75% chance to turn out males when the average temperature of the nest over the incubation period is 29°C. The following experiment is carried out in the wild.

(a) Twenty (20) nests are independently selected at random, among nests that experienced a temperature of 29°C on average over their incubation period. From each nest, a single egg is selected at random. Let $Y$ be the number of eggs that turn out females. Determine the probability that two eggs or more (among the 20 sampled eggs) produce a female turtle.

(b) Now assume that 140 eggs are sampled as before: 140 nest are selected at random among nests that experienced 29°C on average, and a single egg is selected at random from each nest.

i. Let $Y$ be the number of eggs in the sample that turn out females. Calculate the expectation and variance of $Y$.

ii. Calculate the probability that at most 20% eggs in the sample produce female turtles. In other words, calculate $\Pr\{28$ or fewer eggs produce females}. If you make assumptions, justify them.
5. An arborist is assigned the task of planting a long row of 50 elm trees along the side of a road. He has two candidate species, which we will call A and B. He knows that species A grows to an average height of 37m, with variance $70m^2$, and that species B grows to an average height of 38m, with variance $25m^2$, but he does not know the exact distribution of the heights.

(a) The arborist would prefer that the average height of those 50 trees (at maturity) would be greater than 40m. Which species has a higher probability of achieving this goal? Use the CLT to compute the probability for each species, and compare them.

(b) What assumptions did you have to make in part (a) in order to complete the problem? Do they seem reasonable in this situation?