

Record all answers in the blue books. Show your work.

1. (a) Name three types of numerical error that can occur when using fixed-precision representations of real numbers or when doing fixed-precision arithmetic.  
(b) Calculating all pairwise distances among  $n$  objects requires  $n(n-1)/2$  distance computations. Why can we say that this calculation is  $O(n^2)$ ?  
(c) What does it mean for the sequence of discrete-valued random variables  $X_1, X_2, \dots, X_n$  to be a Markov chain?
2. Consider a 3-state Markov chain  $X_1, X_2, \dots, X_n$  based on the transition matrix

$$\begin{bmatrix} 0 & 2/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \\ 1/3 & 2/3 & 0 \end{bmatrix}$$

- (a) The random variable  $X_1$  has a uniform distribution  $(1/3, 1/3, 1/3)$  on the three states. What is the marginal distribution of  $X_2$ ?
- (b) Is the distribution  $(1/4, 1/2, 1/4)$  a stationary distribution of the Markov chain?
3. Consider numbers  $y_1, y_2, \dots, y_n$  and the function  $g(x) = \sum_{i=1}^n (y_i - x)^2$ . By differentiation, show that  $g(x)$  is minimized at  $x = \frac{1}{n} \sum_{i=1}^n y_i$ .
4. One trial of an experiment produces an unordered pair of tickets sampled randomly from a large urn in which a proportion  $\theta$  of tickets have label  $A$ , and a proportion  $1 - \theta$  have label  $a$ . What probabilities are associated with the three possible outcomes? In  $n = 8$  independent and identical trials, we observe 4  $AA$ 's, 4  $aA$ 's, and 0  $aa$ 's. Write the likelihood for  $\theta$  and obtain the maximum likelihood estimate  $\hat{\theta}$ .