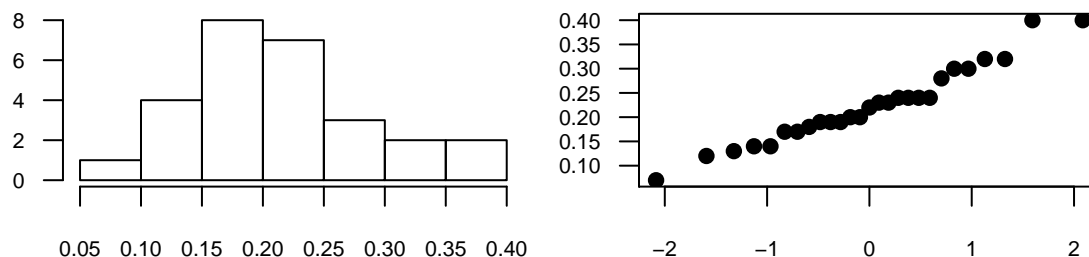


The left ventricular ejection fraction (LVEF) measures the fraction of blood ejected from the left ventricle per heart beat. In subjects with enlarged hearts, this fraction can be quite low. The following data represents the LVEF measurements from a random sample of 27 subjects with enlarged hearts and is displayed with a histogram and with a normal probability plot.

0.07 0.12 0.13 0.14 0.14 0.17 0.17 0.18 0.19 0.19 0.19 0.20 0.20 0.22 0.23
0.23 0.24 0.24 0.24 0.24 0.28 0.30 0.30 0.32 0.32 0.40 0.40



- Find the lower and upper quartiles and the median of the data.
- A modified boxplot identifies outliers that are more than 1.5 IQR (interquartile range) units away from the quartiles as potential outliers. Display the data with a modified boxplot.
- Just to be different, find a 98% confidence interval for the mean LVEF in this population. Interpret the confidence interval in the context of the problem.
- Is your confidence interval valid? Comment on the features of the data apparent in the numerical and graphical summaries that might affect the validity of the confidence interval in the previous part.
- Find a 95% confidence interval for the proportion of subjects in the population whose LVEF measurements are less than 0.15. Interpret the confidence interval in the context of the problem.

Solution:

- The sample size is $n = 27$ so the median is the $27/2 \uparrow 14$ th sorted observation and the quartiles are $27/4 \uparrow 7$ th observations from each end. The lower quartile is 0.17, the median is 0.22, and the upper quartile is 0.28.
- The IQR is $0.28 - 0.17 = 0.11$, so the fences are $(0.11)(1.5) = 0.165$ from each quartile, namely at $0.17 - 0.165 = 0.005$ and $0.28 + 0.165 = 0.445$. There are no observations outside the fences. Whiskers extend to the extreme values within the fences, namely to 0.07 and 0.40. The boxplot must be accompanied by a numerical axis with a regular scale.
- There are 26 degrees of freedom, the mean is 0.224, and the standard deviation is 0.080. The t multiplier is 2.479. The confidence interval is

$$0.224 \pm 2.479 \frac{0.080}{\sqrt{27}} \quad \text{or} \quad 0.224 \pm 0.038$$

In the context of the problem,

We are 98% confident that the mean LVEF in this population of subjects with enlarged hearts is between 0.186 and 0.262.

- If we take the random sampling information at face value, we need be concerned only with evidence of non-normality substantial enough to invalidate the methods. The histogram and normal scores plot both indicate a very small amount of skewness, but the sample size should be large enough to overcome this.

- (e) The proportion of sample individuals with LVEF measurements less than 0.15 is $5/27$, so the modified proportion is $7/31 \doteq 0.225$. The estimated SE is $\sqrt{(7/31)(24/31)/31} \doteq 0.0751$. The 95% confidence interval is 0.225 ± 0.146 . In the context of the problem,

We are 95% confident that the proportion of individuals in the population of subjects with enlarged hearts whose LVEF is less than 0.15 is between 0.079 and 0.371.