Mutations in the hemoglobin, beta gene (HBB) can cause the condition sickle cell anemia if an individual inherits defective alleles from both parents. Individuals with genotypes BB and Bb do not have the condition, but individuals with genotype bb will have sickle cell anemia. Individuals with genotype Bb are called *carriers*. If one or both parents have genotype BB, all children will not have sickle cell anemia. If both parents are carriers, there is a 0.25 probability that each independent child will have sickle cell anemia.

The distribution of genotypes depend on the parent genotypes as follows:

	Child Genotype		
Parent Genotypes	BB	Bb	bb
$BB \times BB$	1	0	0
$BB \times Bb$	1/2	1/2	0
$Bb \times Bb$	1/4	1/2	1/4

PARENT 1 is known to be a carrier. Based on a pedigree analysis, PARENT 2 has a probability 0.4 of being a carrier and a probability 0.6 of having genotype BB. These parents have three children. Assume genotypes of the children are independent given the genotypes of the parents. Express all answers as decimals with at least three significant figures.

(a) If both parents are carriers, what is the probability that *exactly two* of the three children are carriers?

(b) If both parents are carriers, what is the probability that none of the children have sickle cell anemia?

(c) What is the probability that none of three children have sickle cell anemia?

(d) Given that all three children do not have sickle cell anemia, what is the probability that PARENT 2 is a carrier?

Solution:

(a) Let $Y = \#$ of carriers, so $Y \sim \text{Bin}(3, 0.5)$.

$$\mathbb{P}(Y = 2) = \frac{3!}{2!1!}(0.5)^2(0.5)^1 \doteq 0.375$$

(b) Now let $Y = \#$ with sickle cell anemia, so $Y \sim \text{Bin}(3, 0.25)$.

$$\mathbb{P}(Y = 0) = \frac{3!}{0!3!}(0.25)^0(0.75)^3 \doteq 0.422$$

(c) Let $Y = \#$ with sickle cell anemia, so $Y \sim \text{Bin}(3, 0.25)$ conditionally if Parent 2 is a carrier whereas $\mathbb{P}(Y = 0) = 1$ if Parent 2 is not a carrier. Use the law of total probability to find $\mathbb{P}(Y = 0)$.

$$\begin{aligned} \mathbb{P}(Y = 0) &= \mathbb{P}(Y = 0 \mid P_2 \text{ is not a carrier})\,\mathbb{P}(P_2 \text{ is not a carrier}) \\ &\quad + \mathbb{P}(Y = 0 \mid P_2 \text{ is a carrier})\,\mathbb{P}(P_2 \text{ is a carrier}) \\ &= (1)(0.6) + (0.422)(0.4) \\ &\doteq 0.769 \end{aligned}$$

(d) Use Bayes' Theorem to find $\mathbb{P}(P_2 \text{ is a carrier} \mid Y = 0)$.

$$\begin{aligned} \mathbb{P}(P_2 \text{ is a carrier} \mid Y = 0) &= \frac{\mathbb{P}(P_2 \text{ is a carrier and } Y = 0)}{\mathbb{P}(Y = 0)} \\ &= \frac{(0.4)(0.422)}{0.769} \\ &\doteq 0.22 \end{aligned}$$