Suppose that in a population of American adult coffee drinkers, the change in measured heart rate ten minutes after drinking 6 ounces of coffee is normally distributed with a mean increase of 7.3 beats per minute and a standard deviation of 11.1 beats per minute. A negative change would indicate a decrease in heart rate. In answering the following questions, ignore the reality that heart rate measurements are discrete (number of beats in a fixed time interval) and imagine that we used instead a more sophisticated form of measurement that gives continuous values.

(a) What proportion of individuals in the population would have a measured decrease in heart rate?

Solution: The answer is the proportion of individuals with measurements below 0. Measurements are normal with $\mu = 7.3$ and $\sigma = 11.1$. Let $Y$ be the measurement of an individual.

$$P(Y < 0) = P\left(\frac{Y - 7.3}{11.1} < \frac{0 - 7.3}{11.1}\right) = P(\bar{Z} < -0.66) = 0.2546$$

(b) The middle 90 percent of individuals have measured changes between what two values?

Solution: From the table, the 0.95 quantile is between 1.64 and 1.65. I will use 1.645. Ninety percent of individuals have measurements between $7.3 - 1.645(11.1) = -11$ and $7.3 + 1.645(11.1) = 25.6$.

(c) In a random sample of eight adult American coffee drinkers, what is the probability that the sample mean change in heart rate would be below zero?

Solution: The sampling distribution of $\bar{Y}$ is normal with mean 7.3 and standard deviation $\frac{11.1}{\sqrt{8}} = 3.924$. The area to the left of zero is the following.

$$P(\bar{Y} < 0) = P\left(\frac{\bar{Y} - 7.3}{3.924} < \frac{0 - 7.3}{3.924}\right) = P(\bar{Z} < -1.86) = 0.0314$$

(d) In a random sample of eight individuals from the population, what is the probability that exactly seven of eight individuals in the sample have increases in their measured heart rates?

Solution: If $X$ is the number of individuals in the sample whose heart rate increases, then $X$ has a binomial distribution with $n = 8$ and $p = 1 - 0.2546 = 0.7454$.

$$P(X = 7) = \frac{8!}{7!1!}(0.7454)^7(0.2546)^1 = 0.2604$$
(e) Use the normal approximation to the binomial distribution to estimate the probability that there would be 160 or more individuals whose heart rate increased in a random sample of 200 individuals from the population.

Solution: If the sample size is 200, then $X$ is binomial with $n = 200$ and $p = 0.7454$. The mean is $\mu = 149.08$ and the standard deviation is $\sigma = 6.1608$. The exact answer is

$$P(X \geq 160) = \sum_{k=160}^{200} \frac{200!}{k!(200-160)!} (0.7454)^k (0.2546)^{200-k} = 0.0428$$

but you need a computer to get this answer. The normal approximation (with the correction for continuity) is the area to the right of 159.5. If $Y$ is normal with $\mu = 149.08$ and $\sigma = 6.1608$,

$$P(Y > 159.5) = P \left( \frac{Y - 149.08}{6.1608} > \frac{159.5 - 149.08}{6.1608} \right)$$

$$= P(Z > 1.69)$$

$$= 0.0455$$

Without the continuity correction, this would be as follows.

$$P(Y > 160) = P \left( \frac{Y - 149.08}{6.1608} > \frac{160 - 149.08}{6.1608} \right)$$

$$= P(Z > 1.77)$$

$$= 0.0384$$