The purple pigment anthocyanin causes flower petals to be purple. The biochemical pathway for the synthesis of anthocyanin from a colorless precursor requires three enzymes — if any of the three enzymes is not produced, then the biochemical pathway is broken, no purple pigment is produced, and flower color is white. Each of the three enzymes is responsible for one step of the process and is controlled by a single gene with multiple alleles, some of which produce an effective enzyme and some of which do not.

We simplify the problem by considering genetic crosses among individuals that all produce one of the enzymes, but have multiple alleles for the other two genes. Genes $C$ and $P$ each have two alleles ($C$ and $c$, $P$ and $p$). Individuals with genotypes $CC$ or $Cc$ produce enzyme $C$, but individuals with genotype $cc$ do not. For gene $P$, the situation is identical — only the homozygous recessive genotype $pp$ fails to produce enzyme $P$. You may assume that the two genes are inherited independently. An individual plant with both enzymes has purple flowers, while an individual lacking one enzyme or both has white flowers.

The exam questions are all based on the following set of genetic crosses. In the first cross, INDIVIDUAL A with genotype $CCPp$ is crossed with INDIVIDUAL B with genotype $ccPp$. Standard genetic theory shows that the offspring can have three possible genotypes, $CcPP$, $CcPp$, and $Ccpp$ with probabilities $1/4$, $1/2$, and $1/4$, respectively. In a second cross, a randomly selected offspring from the first cross, INDIVIDUAL C, is crossed with INDIVIDUAL D with genotype $Ccpp$ producing INDIVIDUAL E.

In solving each of the following problems, present sufficient work so that your method of solution is clear. Clearly define any notation for events you introduce for the problem. (For example, you might specify the event $E_{\text{purple}} = \{\text{INDIVIDUAL E has purple flowers}\}$.)

Please use the space below and on the back of the page to show your exam work.

(a) If INDIVIDUAL C has genotype $CcPP$, what is the probability that INDIVIDUAL E can produce enzyme $C$?
(b) If INDIVIDUAL C has genotype $CcPp$, what is the probability that INDIVIDUAL E has white flowers?
(c) What is the probability that INDIVIDUAL E has white flowers?
(d) What is the probability that INDIVIDUAL E has white flowers given that INDIVIDUAL C has purple flowers?
(e) What is the probability that INDIVIDUAL C has purple flowers given that INDIVIDUAL E has white flowers?

Solution: A tree diagram is useful to guide these calculations.
(a) E can produce enzyme C if its genotype for gene C is either CC or Cc. Each of its parents has genotype Cc. We have that

\[ P(\text{E produces } C | \text{C is } Cc) = 1 - P(\text{E does not produce } C | \text{C is } Cc) \]
\[ = 1 - P(\text{C gives } c \text{ and } D \text{ gives } c | \text{C is } Cc) \]
\[ = 1 - (1/2)(1/2) \]
\[ = 3/4 \]

(b) E has white flowers if it cannot produce enzyme C or it cannot produce enzyme P. Let \( E_W \) be the event that E has white flowers and \( E_P \) be the event that E has purple flowers.

\[ P(\text{E is } CcPp | \text{C is } CcPp) = 1 - P(\text{E is } E_P | \text{C is } CcPp) \]
\[ = 1 - P(\text{E has enzyme C and E has enzyme P} | \text{C is } CcPp) \]
\[ = 1 - P(\text{E has enzyme C} | \text{C is } CcPp) \times P(\text{E has enzyme P} | \text{C is } CcPp) \]
\[ = 1 - (3/4)(1/2) \]
\[ = 5/8 \]

(c) The probability that E has white flowers is a weighted average of the conditional probabilities of white flowers weighted by the probabilities of the genotype of C.

\[ P(\text{E is white}) = (1/4)(1/4) + (1/2)(5/8) + (1/4)(1) = 5/8 \]

(d) We find the probability that E has white flowers given that C has purple flowers by using the rule for conditional probabilities. Let \( C_P \) be the event that Individual C has purple flowers.

\[ P(\text{E is } CcPp | \text{C is } CcPp) = \frac{P(\text{E is } CcPp \text{ and } C_P)}{P(C_P)} \]
\[ = \frac{(1/4)(1/4) + (1/2)(5/8)}{3/4} \]
\[ = 1/2 \]

(e) Nominally, this is Bayes' Theorem, but we can use previous work to answer it.

\[ P(\text{C is } CcPp | \text{E is white}) = \frac{P(\text{C is } CcPp \text{ and } E_W)}{P(\text{E is white})} \]
\[ = \frac{(1/4)(1/4) + (1/2)(5/8)}{5/8} \]
\[ = 3/5 \]