The Poisson distribution arises in many biological contexts. Examples of random variables for which a Poisson distribution might be reasonable include:

- the number of bacterial colonies in a Petri dish;
- the number of trees in an area of land;
- the number of offspring an individual has;
- the number of nucleotide base substitutions in a gene over a period of time;

The Poisson distribution is discrete, like the binomial distribution, but has only a single parameter $\mu$ that is both the mean and the variance.

In R, you can compute Poisson probabilities with the function `dpois`. For example, if $\mu = 10$, we can find $\Pr\{Y = 12\} = e^{-10} \frac{10^{12}}{12!}$ with the command

```r
dpois(12, 10)
```

This approximation is most useful when $n$ is large so that the binomial coefficients are very large.
Example (cont.)

Find the probability of three or more individuals.

Solution: Instead of summing the probabilities from 3 to infinity, we can use the complement rule.

\[ P(Y \geq 3) = 1 - P(Y = 0) - P(Y = 1) - P(Y = 2) \]

In R, this is found by one of two ways.

```r
> 1 - ppois(2, 1.8)
[1] 0.2693789
> 1 - sum(dpois(0:2, 1.8))
[1] 0.2693789
```

The Poisson Process

- The Poisson Process arises naturally under assumptions that are often reasonable.
- For the following, think of points as being exact times or locations.
- The assumptions are:
  - The chance of two simultaneous points is negligible;
  - The expected value of the random number of points in a region is proportional to the size of the region.
  - The random number of points in non-overlapping regions are independent.
- Under these assumptions, the random variable that counts the number of points has a Poisson distribution.
- If the expected rate of points is \( \lambda \) points per unit length (area), then the distribution of the number of points in an interval (region) of size \( t \) is \( \mu = \lambda t \).

Example

Suppose that we assume that at a location, a particular species of plant is distributed according to a Poisson process with expected density 0.2 individuals per square meter. In a nine square meter quadrat, what is the probability of no individuals?

Solution: The number of individuals has a Poisson distribution with mean \( \mu = 9 \times 0.2 = 1.8 \). The probability of this is

\[ P(Y = 0 | \mu = 1.8) = \frac{e^{-1.8} (1.8)^0}{0!} = 0.165299 \]

In R, we can compute this as

```r
> dpois(0, 1.8)
[1] 0.1652989
```