Problem 5.34

Let $Y$ be the number of the oral contraceptive users in the random sample of $n$. Then $\hat{p} = Y/n$. The distribution of $Y$ is Binomial($p, n$), and $p = 0.12$. When $n$ is large, $Y$ is approximately distributed as a Normal($np, \sqrt{np(1-p)}$).

(a) When $n = 100$, $np = 12$ and $\sqrt{np(1-p)} = \sqrt{100 \times 0.12 \times (1-0.12)} = 3.25$

\[
\Pr \{ p - 0.03 \leq \hat{p} \leq p + 0.03 \} = \Pr \{ 0.09 \leq \hat{p} \leq 0.15 \} = \Pr \{ 9 \leq Y \leq 15 \} = \Pr \{ 8.5 \leq Y \leq 15.5 \} \quad \text{(Continuity Correction)}
\]
\[
= \Pr \left\{ \frac{8.5 - 12}{3.25} \leq Z \leq \frac{15.5 - 12}{3.25} \right\}
\]
\[
= \Pr \{ -1.08 \leq Z \leq 1.08 \} = 0.8599 - 0.1401 = 0.7198
\]

(b) $n = 200$, $np = 24$, and $\sqrt{np(1-p)} = \sqrt{200 \times 0.12 \times (1-0.12)} = 4.60$

\[
\Pr \{ p - 0.03 \leq \hat{p} \leq p + 0.03 \} = \Pr \{ 0.09 \leq \hat{p} \leq 0.15 \} = \Pr \{ 18 \leq Y \leq 30 \} = \Pr \{ 17.5 \leq Y \leq 30.5 \} \quad \text{(Continuity Correction)}
\]
\[
= \Pr \left\{ \frac{17.5 - 24}{4.6} \leq Z \leq \frac{30.5 - 12}{4.6} \right\}
\]
\[
= \Pr \{ -1.41 \leq Z \leq 1.41 \} = 0.9207 - 0.0783 = 0.8414
\]