Practice problems from the textbook

- Exercises 6.56 and 6.57.
- Exercises 7.86, 7.87, 7.88, 7.96 and 7.98 (a).
- Exercises 9.43, 9.44, 9.45, 9.46 (a), 9.49 and 9.51.
- Exercises 10.76, 10.81
- Exercises 11.40, 11.41, 11.42 and 11.37.
- Exercises 12.45, 12.46, 12.47, 12.53 and 12.55.
- Exercises 13.6, 13.7, 13.9, 13.11 and 13.14. I encourage you to try out more problems from Chapter 13, especially those about choosing the appropriate method and where no calculation is required.

Problems from past exams

1. A scientist wishes to compare two methods for removing atrazine contamination from the soil. Suppose he has available 8 contaminated plots of soil, each a square 10 meters on a side. Of the 8 plots, 4 are randomly assigned to receive method A and 4 to receive method B. On each plot, 3 (independent) readings are obtained. The following are the observed concentrations of atrazine in parts per million for each reading on each plot.

Method A				Method B			
Plot 1	Plot 2	Plot 3	Plot 4	Plot 5	Plot 6	Plot 7	Plot 8
4.9	6.5	5.5	5.8	6.5	7.2	5.6	5.9
5.2	6.3	5.0	5.7	6.0	7.0	5.4	6.3
4.9	6.4	5.4	5.6	6.4	7.1	5.5	6.1

   The scientist plans on analyzing the data with an independent sample t-test, by pooling the 12 observations on each method together as one sample for each method. Is this analysis correct? Justify briefly.

2. Thrombolytic drugs are substances that cause the breakdown of blood clots obstructing the flow of blood through the vessels. They are used by injection during or shortly after a heart attack or stroke to prevent clots from blocking blood flow to the heart muscle or brain. In an experiment, three thrombolitic treatments were studied: streptokinase (SK), accelerated alteplase (AtPA) and streptokinase plus alteplase (SK+tPA). A total of 4082 patients affected by Acute Myocardial Infarction (AMI) participated in the study. Each patient received one of the drugs, randomly assigned, shortly after the infarction. Efficacy was measured by the mortality at 30 to 35 days.

   (a) The data are summarized in the following table. Test the null hypothesis that the three thrombolitic drugs yield the same probability of survival (use $\alpha = 0.05$).

Drug	SK	AtPA	SK+tPA
Death	147	65	72
Survival	1869	969	960
Total	2016	1034	1032

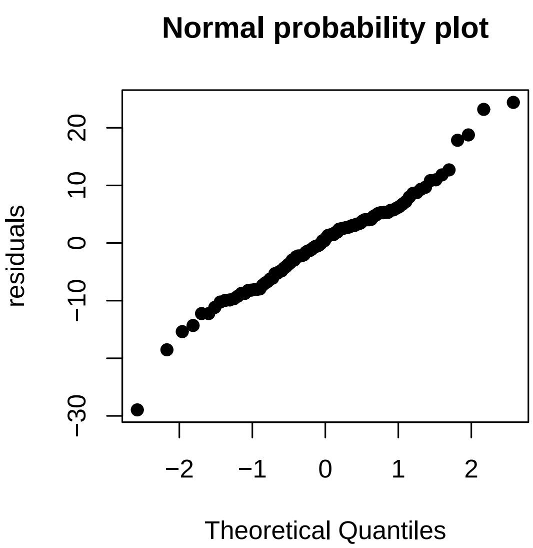
(b) Check the validity of the method applied in 2a.

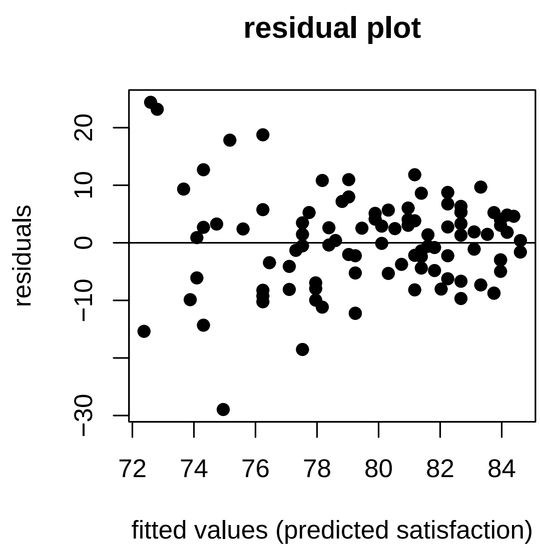
3. An investigation is performed to understand the relationship between age and satisfaction with medical care. Each individual in the investigation is asked to rate satisfaction with medical care on a scale of 0 to 100, with 100 denoting complete satisfaction. Age was also recorded (among other sociodemographic characteristics) on each individual. A total of 100 individuals are involved in the investigation. The linear regression analysis of satisfaction versus age yields satisfaction = 67.6 + 0.21 age, with a correlation coefficient $r = 0.37$ and slope $b_1 = 0.21$ (p-value = 0.0002).

   (a) Are older or younger individuals more likely to be more satisfied with medical care? (be brief).

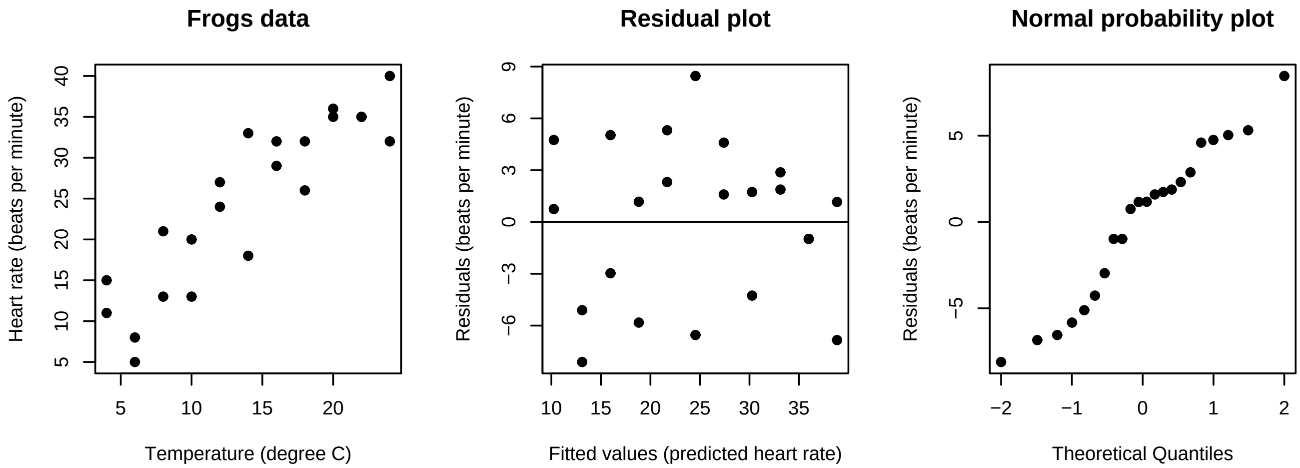
   (b) Is the linear relationship between age and satisfaction statistically significant? Justify shortly.

   (c) Below are a normal probability plot of the residuals and a plot of residuals vs. fitted values (predicted satisfaction). Under what conditions is a linear regression analysis valid? Are these conditions met here?





4. An experimenter was interested in the relationship between temperature and heart rate in the common grass frog. The temperature was manipulated in 2-degree increments ranging from 4 to 24 degrees C with heart rates recorded at each temperature in beats per minute. Two observations were made at selected each temperature, for a total of 22 observations. A different randomly selected frog was used for each observation. A scatter-plot of the data (Y = heart rate versus X = temperature) is given below along with a residual plot and a normal probability plot of the residuals. Calculations yield $\bar{x} = 14$, $\bar{y} = 24.55$, $\sum(x_i - \bar{x})(y_i - \bar{y}) = 1258$, $\sum(x_i - \bar{x})^2 = 880$, SS(total) $= \sum(y_i - \bar{y})^2 = 2237$ and SS(residual) $= 439$.



(a) Calculate the intercept and the slope of the regression line of heart rate (Y) on temperature (X). Draw the regression line on the plot.

(b) What is the predicted value of a new frog's heart rate at 15 °C?

(c) Determine the residual standard deviation s_e (and show your work). It is one of ☐ 4.572 ☐ 4.685 ☐ 4.938. How many frogs in the sample have their heart rate within two s_e from the predicted value (i.e. from the regression line)?

(d) Provide a 95% confidence interval for the slope β_1. Is the slope different from 0 with statistical significance (at $\alpha = .05$)?

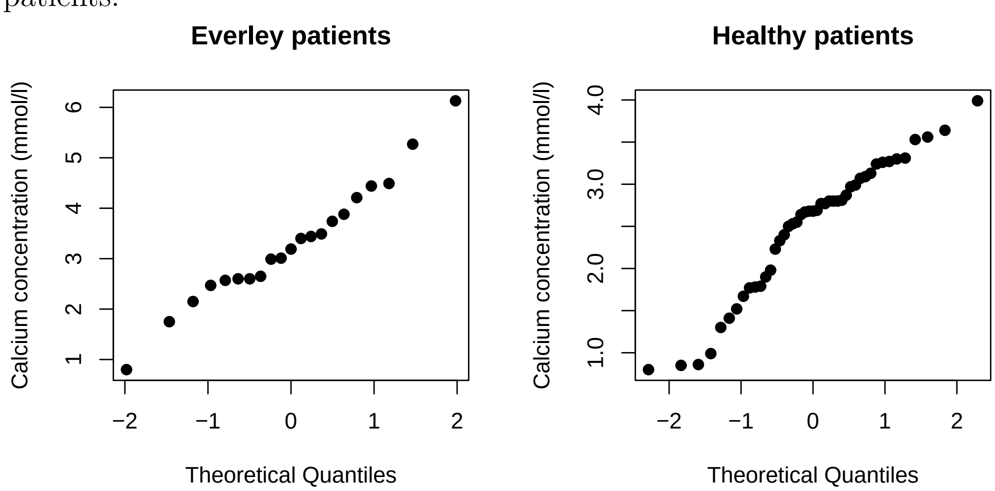
(e) Calculate the correlation coefficient between heart rate and temperature.

(f) Check whether the hypotheses underlying the regression analysis are met here.

5. A rare congenital disease, Everley's syndrome, generally causes a reduction in concentration of blood sodium. A study was conducted to determine if plasma calcium concentration is also affected in patients with Everley's syndrome. Plasma calcium concentration was measured in 21 patients with Everley's syndrome, as well as in 45 healthy patients.

The data are summarized in the following table, and normal probability plots are indicated below.

	Everley	Healthy
Mean $\bar{y}$ (in mmol/l)	3.31	2.50
SD s (in mmol/l)	1.2	0.8
n	21	45



(a) Are these data ☐ observational or ☐ experimental?

(b) Is the plasma concentration in patients with Everley's syndrome abnormally high? State first the name of the test you will use, and let $\alpha = 0.05$. Hint: the degree of freedom is one of ☐ 19.2 ☐ 28.6 ☐ 65.1

(c) Check that the method you used in 5b is valid for the data at hand.

6. A study was conducted to compare the coefficients of digestibility of dry matter for four diets fed to goats. Six randomly selected goats were assigned to each treatment. Data are indicated on the right. Sample means and standard deviations are given below. Also, calculations yield SS(error) = 250.5 and SS(total) = 689.6.

A	B	C	D
51	49	67	57
57	53	57	57
49	50	65	58
54	52	60	60
53	50	56	64
57	44	56	59

Diet	A	B	C	D
mean digestibility	53.5	49.7	60.2	59.2
SD	3.21	3.14	4.79	2.64
n	6	6	6	6

(a) Test the null hypothesis that mean coefficients of digestibility are the same for all 4 diets. State the conclusion of the test, and bracket the p-value.

(b) Is the method used in 6a valid here? (check assumptions!)

(c) Determine the pooled standard deviation s_{pooled} (show your work). It is one of ☐ 3.23 ☐ 3.30 ☐ 3.54

(d) Use the Newman-Keuls method to compare all pairs of means at $\alpha = 0.05$. Is there sufficient evidence to tell which diet has the highest population mean digestibility, and which diet has the lowest mean digestibility? Note: in case you didn't answer question 6c, pick any value of s_{pooled} from 6c in order to carry out the Newman-Keuls method.

Keys to practice problems from the textbook

6.56. $28.86 \pm 2.576 * 4.24/\sqrt{1353}$ i.e (28.56, 29.16) days. 29.5 is not in the interval.

6.57. women with shorter cycles had more cycles during the fixed time period, they contributed more observations to the data.
Not valid: 5412 observations are not independent. Hierarchical structure in the data.

7.86. $\mathrm{SE}_{\bar{y}_1-\bar{y}_2} = \sqrt{16.1^2/38 + 31.7^2/45} = 5.399$ and $t = (\bar{y}_1 - \bar{y}_2)/\mathrm{SE} = (168.2-107.9)/5.399 = 11.2$. Degree of freedom df= 67.5 was given, and p-value < 0.001. Very strong evidence that mean platelet calcium is higher in people with high blood pressure than in people with normal blood pressure.

7.87. Multiplier: (df=70) $t = 1.994$. (=2.000 with df=60). We get $(168.2 - 107.9) \pm 1.994 * 5.399$ i.e (49.5, 71.1) nM.

7.88. No. The t-test is valid because the sample sizes are rather large.

7.96. The lack of statistically significant evidence in weight gain does not show that the new diet is as good as the standard diet. Evidence to that could be obtained from a confidence interval (for instance).

7.98. (a). Two of the patients contributed 2 observations each to the data set. Hierarchical structure. t-test not appropriate.

9.43. Standard error is $1.86/\sqrt{15} = .48$. df=14, we get $2.2 \pm 1.761 * 0.48$ i.e (1.35, 3.05) fishes.

9.44. It must be reasonable to regard the 15 differences as a random sample from a normal population. We must trust the researchers that their sampling method was random. The normality condition can be verified with a normal probability plot. It is fairly linear, which supports the normality condition.

9.45. $t = 2.2/0.48 = 4.58$. We have df= $15 - 1 = 14$ and $p < 0.001$. Very strong evidence that the average number of species in pools is greater than in riffles.

9.46. (a) $B_+ = 12$, $B_- = 1$. We eliminate 2 pairs with $d = 0$. $n_d = 13$ and so $.002 < p < .01$. Strong evidence.

9.49. differences: 9, 20, 3, 2, 8, 6, 6, 2, 10. Mean: 7.33, SD= 5.59 and SD= $5.59/\sqrt{9} = 1.86$. The test statistic is $t = 7.33 - 1.86 = 3.94$. df= 9-1=8 and we get $0.001 <$ p-value < 0.01. There is strong evidence that caffeine tends to decrease RER under these conditions.

9.51. $B_+ = 9$, $B_- = 0$, $n_d = 9$. Therefore, $.002 < p < .01$. Strong evidence again.

10.76 Let p=probability of female, 1=warm, 2=cold.
(a) expected frequencies: female: $0.5*141 = 70.5$, same as the male's. $X^2 = (73-70.5)^2/70.5 + (68 - 70.5)^2/70.5 = 0.18$. There are 2 categories so df=2-1=1. From table 9 we find that p> .20. Insufficient evidence that the sex ratio is not 1:1 in the warm environment.
(b) expected frequencies are now 0.5*169=84.5 for both males and females. $X^2 = (107 - 84.5)^2/84.5 + (62 - 84.5)^2/84.5 = 11.98$. We get $.0001 < p < .001$. Very strong evidence that the cold environment produces more females than males.
(c) observed and expected frequencies:

	warm	cold	total
male	68 (59.13)	62 (70.87)	130
female	73 (81.87)	107 (98.13)	180
total	141	169	310

The χ^2 statistic is $X^2 = 4.20$. We have df=1 so $.02 < p < .05$. To determine directionality we calculate $\hat{p}_1 = 73/141 = .52$ and $\hat{p}_2 = 107/169 = .63$. Moderate evidence that $p_1 > p_2$, i.e. probability of a female is higher in the cold than in the warm environment.
(d) All eggs from one mating. No basis for generalizing to other *Menidia* individuals. The population can be defined as all (potential) offspring of the single mating or perhaps the matings of the same genotypes.

10.81 No. The proposed analysis would not be valid because the observations on cilia from the same child are not independent.

11.40 H_0: the three classes produce the same mean change in fat-free mass.

source	df	SS	MS	F	p-value
class	2	2.465	1.2325	0.64	$> .20$
error	26	50.133	1.9282		
total	28	52.598			

11.41 (a) The F-test is based on the populations having normal distributions with a common standard deviation.
(b) The distributions appear to be reasonably symmetric (although aerobic...) The modern dance group SD appears to be smaller than the other 2 SDs, but not much smaller.

11.42 table

source	df	SS	MS	F	p-value
group	3	129.49	43.16	3.56	$.01 < p < .02$
error	207	2506.8	12.11		
total	210	2636.3			

Strong evidence that the populations have different mean refractive errors.

11.37 $\sqrt{.5842/13} = .212$ is the scale factor.
(a) with df=120, $q_{10} = 4.56$ so $R_{10} = .967$ which is greater than the greatest observed difference: 2.84 (treatment 6)-2.07 (treatment 4)=.77.

$$\underline{T_4 \quad T_2 \quad T_7 \quad T_3 \quad T_{10} \quad T_5 \quad T_8 \quad T_1 \quad T_9 \quad T_6}$$

None of the null hypothesis is rejected, and the procedure stops here. We don't even need to calculate R_9, nor any of $R_8 \ldots R_2$. There is insufficient evidence to say that any of the treatments produce different mean liver weights.
(b) $q_2 = 2.80$ then $R_2 = .594$. Ordering of the treatments:
$T_4 = 2.07 < T_2 = 2.28 < T_7 = 2.29 < T_3 = 2.34 < T_{10} = 2.37 < T_5 = 2.40 < T_8 = 2.45 < T_1 = 2.59 < T_9 = 2.76 < T_6 = 2.84$. The most extreme comparisons are $T_6 - T_4 = .77$, we reject. $T_6 - T_2 = .56$, we do not reject. $T_9 - T_4 = .69$ we reject. $T_1 - T_4 = .52$ we do not reject, and the procedure stops here, because we don't test pairs of means that are already under the same line.

$T_4 \quad T_2 \quad T_7 \quad T_3 \quad T_{10} \quad T_5 \quad T_8 \quad T_1 \quad T_9 \quad T_6$

12.45 (a)regression line: $b_1 = -0.342/0.1512 = -2.262$ and $b_0 = 1.117 + 2.262 * 0.12 = 1.39$.
(c) $s_e = \sqrt{SS(res)/(n-2)} = .17$ kg.

12.46 (a) when $x = .24$ we get $y = 1.39 - 2.262 * 0.24 = 0.85$ kg. SD = 0.17 kg.
(b) The condition that σ does not depend on X is doubtful. More variability in Y when X is small than when X is large. SDs are: .21, .28, .11, .06.

12.47 There are 2 ways to go. SE for the slope $\text{SE}_{b_1} = .1719/\sqrt{.1512} = .4421$. Then $t = -2.262/.4421 = -5.12$ and $p < .001$ (df=12-2=10). The other way is with the F test

source	df	SS	MS	F	p-value
regression	1	0.7735	0.7735	26.17	$.0001 < p < .001$
residual	10	.2955	0.02955		
total	11	1.069			

12.53 (a) The 3 correlations are: 0.94 (men), 0.81 (women) and 0.08 (both).
(c) $b_1 = .02069$ and $b_0 = 26.9$.
(d) For each sex, heavier people tend to have a higher percent fat. But women tend to lighter than men and to have a higher percent fat than men. These tendencies approximately cancel out.

13.6 paired-sample t-test, CI, or sign test if not normally distributed.

13.7 Analysis of variance.

13.9 two-independent-sample comparison. But not normally distributed. Wilcoxon-Mann-Whitney test instead.

13.11 correlation and regression.

13.14 χ^2 test of independence.

Keys to past exams' problems

1. No: the 12 observations are not independent.

2. (a) $X^2 = 1.03$ $df = 2$ and $p > .20$. H_0 is accepted. (b) Design seems okay. Expected counts are ≥ 5.

3. (a) older, because slope $b_1 > 0$. (b) Yes because $p < .001$. It shows $\beta_1 > 0$. (c) all assumptions met expect that σ_e is not constant.

4. (a) $b_1 = 1.43$, $b_0 = 4.53$. (b) 26^{o}C. (c) 4.685. $2s_e = 9.37$ so all frogs (i.e. 22) have their heart rate within 2 s_e from the predicted value. (d) $\text{SE}_{b_1} = 0.158$ and $t = 2.086$, we get $(1.10, 1.76)$ beats/^{o}C. Yes, $beta_1 > 0$ with statistical significance because the interval does not include 0. (e) .896 (f) everything okay

5. (a) observational (b) df = 28.6. (other possibilities can be eliminated because 19.2 is $< 21 - 1$ and 65.1 is $> 45 - 1$. Independent sample t-test, non-directional. $\text{SE}_{\bar{y}_1 - \bar{y}_2} = 0.288$, $t = 2.77$ and $.001 < p < .01$. H_0 is rejected. Yes, plasma calcium concentration is abnormally high in Everley patients. (c) everything okay. (healthy sample large enough)

6. H_0 is rejected. All diets do not have the same mean digestibility.

	df	SS	MS	F	p-value
diet	3	439.1	146.4	11.68	$.0001 < p < .001$
residual	20	250.5	12.52		
total	23	689.6			

(b) yes. (randomness, SD about the same, normal probanility plots not available, but there is no outlier) (c) 3.54 (d) scale factor 1.44.

i	2	3	4
q_i	2.95	3.58	3.96
R_i	4.263	5.174	5.723

We get

<u>B A</u> <u>D C</u>

A and B are not significantly different. C and D neither. Most: either C or D (or both). Least: either A or B (or both).

I hope you will do an excellent job!