Statistics 333: Applied Regression Analysis

January 25, 2007

Announcements:

- Some course materials will be uploaded in the course website soon. http://www.stat.wisc.edu/Department/Courses/2006.2.html
- The issue with the course enrollment will be handled soon.
- Any other issues? Please let Prof. Johnson know your issues by email rich@stat.wisc.edu.

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Agenda for Today's Lecture:

Plan to cover:

1. Overview for Simple Regression
2. Introduction to R

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Regression Analysis:

Regression analysis concerns the study of relationships between variables with the objectives of

1. identifying the relationship
2. estimating the parameters
3. validating the estimates
4. predicting the response variable

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Regression with a Single Predictor:

Notation:

- x: independent variable, predictor variable or input variable
- y: dependent variable or response variable

Regression with a Single Predictor:

Example:

Dosage x (in Milligrams) and the Number of Days of Relief y from Allergy for Ten Patients.

Dosage x	Duration of Relief y
3	9
3	5
4	12
5	9
6	14
6	16
7	22
8	18
8	24
9	22

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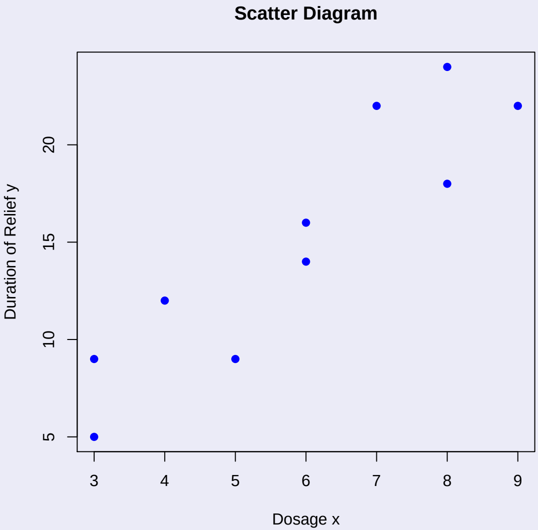
What would you do as a first step to analyze the relationship?

Regression with a Single Predictor:

Scatter Diagram:



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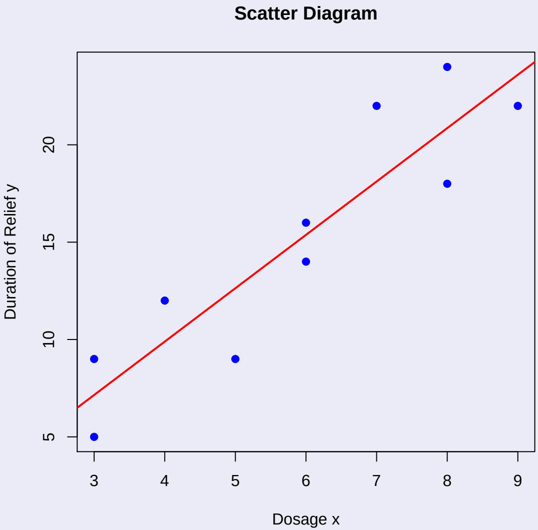


Straight Line Regression Model:

Statistical Model for a Straight Line Regression:

We assume that the response Y is a random variable that is related to the input variable x by

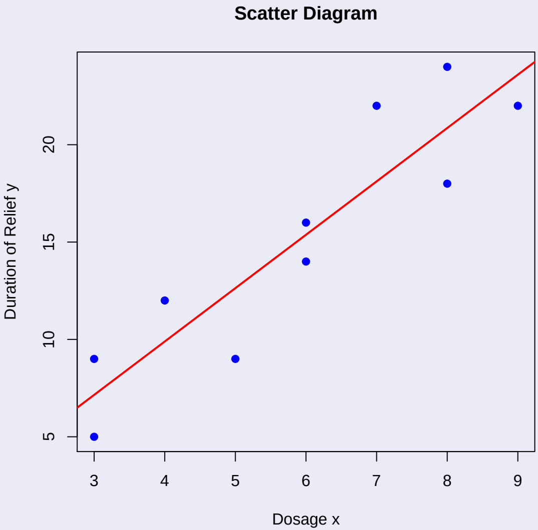
$$Y_i = \beta_0 + \beta_1 x_i + e_i, \quad i = 1, 2, \ldots, n$$

where

1. Y_i denotes the response corresponding to the ith experimental run in which the input variable x is set at the value x_i.
2. $e_1, \ldots, e_n$ are the unobservable random errors which we assume are independently and normally distributed with mean zero and an unknown standard deviation σ.
3. The parameters β_0 and β_1, which together locate the straight line, are unknown.

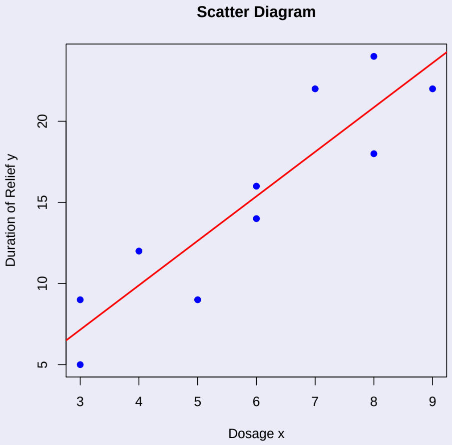
Straight Line Regression Model:

The statistical model for a straight line regression:



Straight Line Regression Model:

Assume the previous statistical model is correct. Then how to fit the best straight line of the y to x relationship in the scatter diagram?



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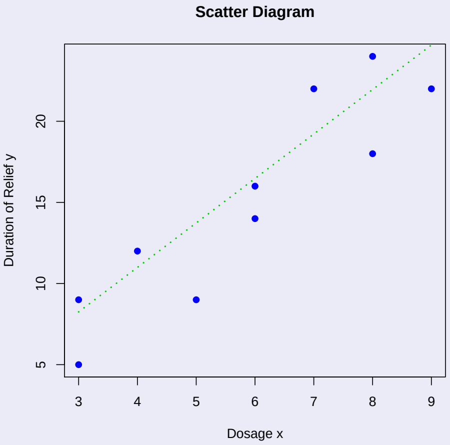

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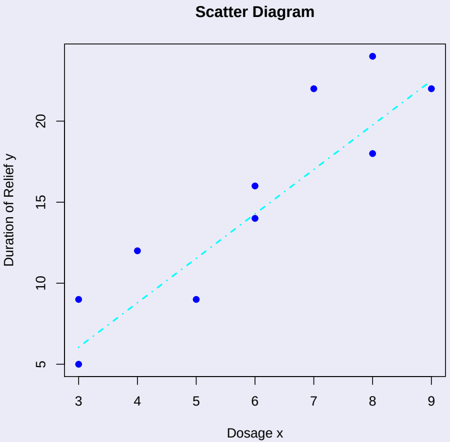
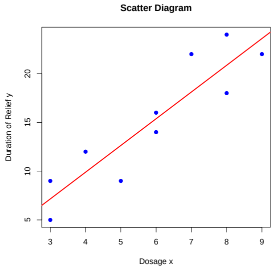


Method of Least Squares:

Suppose that an arbitrary line $y = b_0 + b_1 x$ is drawn on the scatter diagram.



Principle of Least Squares:

Determine the values for the parameters so that the overall discrepancy

$$D = \sum_{i=1}^{n} d_i^2 = \sum_{i=1}^{n} (y_i - b_0 - b_1 x_i)^2$$

is minimized. The parameter values thus determined are called the least squares estimates.

Method of Least Squares:

- Least Squares Estimator of β_0:

$$\hat{\beta}_0 = \overline{y} - \hat{\beta}_1 \overline{x}$$

- Least Squares Estimator of β_1:

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{n}(x_i - \overline{x})(y_i - \overline{y})}{\sum_{i=1}^{n}(x_i - \overline{x})^2}$$

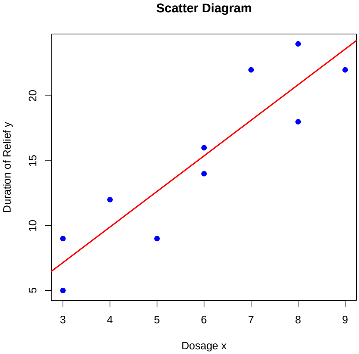
- Fitted (or estimated) regression line:

$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x$$

- Residuals:

$$\hat{e}_i = y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i, \quad i = 1, \ldots, n$$

Method of Least Squares:



$$\hat{y} = -1.071 + 2.741x$$

By using the fitted regression line, we can predict a response y for a specified x value.

Inferences for β_1 and β_0:

```
> Summary(lmfit)

Call:
lm(formula = y ~ x)

Residuals:
    Min      1Q  Median      3Q     Max 
-3.6333 -2.0128 -0.3741  2.0428  3.8851 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  -1.0709     2.7509  -0.389 0.707219    
x             2.7408     0.4411   6.214 0.000255 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1 

Residual standard error: 2.821 on 8 degrees of freedom
Multiple R-Squared: 0.8284,	Adjusted R-squared: 0.8069 
F-statistic: 38.62 on 1 and 8 DF,  p-value: 0.0002555
```

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