

1. Let $Z_1, Z_2, \dots, Z_n, Z_{n+1}$ be independent standard normal random variables.

(a) Determine the distribution of

$$\frac{(Z_1 + Z_2) / \sqrt{2}}{\sqrt{(Z_3^2 + Z_4^2 + Z_5^2) / 3}}$$

(b) Let $\bar{Z} = \sum_{i=1}^n Z_i / n$. Determine the distribution of

$$\frac{(Z_{n+1} - \bar{Z}) / \sqrt{(1 + \frac{1}{n})}}{\sqrt{\sum_{i=1}^n (Z_i - \bar{Z})^2 / (n - 1)}}$$

2. Let X_1, X_2, \dots, X_n be a random sample of size n from the negative exponential distribution having probability density function

$$\frac{1}{\theta} e^{-\frac{x}{\theta}} \quad \text{for } x > 0.$$

Obtain the likelihood ratio test for testing $H_0 : \theta = \theta_0$ versus $H_1 : \theta \neq \theta_0$.

3. Let X_1, X_2, \dots, X_n be a random sample of size n from the Bernoulli distribution where

$$p = P[X_1 = 1] = 1 - P[X_1 = 0]$$

(a) Obtain the likelihood ratio test for testing $H_0 : p = p_0$ versus $H_1 : p \neq p_0$.

(b) With $\alpha = .03$, use the test in Part (a) to test $H_0 : p = .6$ versus $H_1 : p \neq .6$ when 37 out of $n = 81$ drivers report receiving a ticket for a parking violation last year.

The remaining problem is from the book.

4. Exercise 40 page 344