HW3 **Stat 312**

1. Consider a random sample $X_1\, X_2\,, ..., X_n$ of size n from the binomial$(2, p)$ population which has distribution

$$P[\,X\,=\,x\,;\,p\,] = f(x\,;\,p\,] = \begin{pmatrix} 2 \\ x \end{pmatrix} p^x\,(\,1\,-\,p\,)^{2-x} \quad \text{for} \;\; x = 0, 1, 2$$

   (a) Find the maximum likelihood estimator $\widehat{p}$ of p.

   (b) Evaluate the Fisher Information $I_1(\,p,\,)$ in terms of the unknown p.

   (c) If $n = 80$ and $\sum_{i=1}^{60} = 130$ obtain the maximum likelihood estimate of p and give its estimated standard error using your result in Part (b).

2. Consider a random sample $X_1\, X_2\,, ..., X_n$ of size n from the negative exponential distribution, parameterized by θ, so the pdf is $f(x,\,;\,\theta\,)\;=\;\frac{1}{\theta}\,e^{-\,x\,/\,\theta}$ $, x > 0$

   (a) Find the maximum likelihood estimator $\widehat{\theta}$ of θ.

   (b) Evaluate the Fisher Information $I_1(\,\theta)$, in terms of the unknown θ using both

$$I_1(\theta) = E_\theta[(\frac{\partial}{\partial\theta}(\ln f(X_1;\theta))^2\;]$$

   and also

$$E[-\frac{\partial^2}{\partial\theta^2}\ln f(X_1|\theta)\;]$$

   (c) The negative exponential distribution may apply to the life length of a fully charged camcorder battery. Suppose $n = 100$ different fully charged batteries are tested and the total of the lifetimes is $\sum_{i=1}^{100}\, x_i \;=\; 250$ hours. Obtain the maximum likelihood estimator of θ and estimate it's standard error.

(d) Set an approximate large sample 95% confidence interval for θ using Part c.

3. Referring to problem 3, find the maximum likelihood estimate of

$$P[X_1 > 3.0] = e^{-3.0\,/\,\theta}$$