1. Consider a random sample $X_1, X_2, \ldots, X_n$ of size $n$ from the binomial$(2, p)$ population which has distribution

$$P[X = x ; p] = f(x ; p) = \binom{2}{x} p^x (1 - p)^{2-x} \text{ for } x = 0, 1, 2$$

(a) Find the maximum likelihood estimator $\hat{p}$ of $p$.

(b) Evaluate the Fisher Information $I_1(p, )$ in terms of the unknown $p$.

(c) If $n = 80$ and $\sum_{i=1}^{60} = 130$ obtain the maximum likelihood estimate of $p$ and give its estimated standard error using your result in Part (b).

2. Consider a random sample $X_1, X_2, \ldots, X_n$ of size $n$ from the negative exponential distribution, parameterized by $\theta$, so the pdf is $f(x, ; \theta) = \frac{1}{\theta} e^{-x/\theta}$, $x > 0$

(a) Find the maximum likelihood estimator $\hat{\theta}$ of $\theta$.

(b) Evaluate the Fisher Information $I_1(\theta)$, in terms of the unknown $\theta$ using both

$$I_1(\theta) = E_\theta[(\frac{\partial}{\partial \theta} (\ln f(X_1; \theta))^2 ]$$

and also

$$E[- \frac{\partial^2}{\partial \theta^2} \ln f(X_1|\theta) ]$$

(c) The negative exponential distribution may apply to the life length of a fully charged camcorder battery. Suppose $n = 100$ different fully charged batteries are tested and the total of the lifetimes is $\sum_{i=1}^{100} x_i = 250$ hours. Obtain the maximum likelihood estimator of $\theta$ and estimate its standard error.
(d) Set an approximate large sample 95% confidence interval for $\theta$ using Part c.

3. Referring to problem 3, find the maximum likelihood estimate of

$$P[X_1 > 3.0] = e^{-3.0/\theta}$$