An investigator conducted an experiment to compare a new treatment with a control treatment on the germination and growth of seeds, resulting in the data below. Use a chi-square test of independence to test if the new treatment differs form the control.

	No germination	Growth/no seed production	Produces seed
Control	11	17	72
Treatment	5	5	90

The following table displays quantiles from several chi-square distributions.

df	0.8	0.9	0.95	0.96	0.97	0.98	0.99	0.995	0.999	0.9995	0.9999
1	1.64	2.71	3.84	4.22	4.71	5.41	6.63	7.88	10.83	12.12	15.14
2	3.22	4.61	5.99	6.44	7.01	7.82	9.21	10.60	13.82	15.20	18.42
3	4.64	6.25	7.81	8.31	8.95	9.84	11.34	12.84	16.27	17.73	21.11
4	5.99	7.78	9.49	10.03	10.71	11.67	13.28	14.86	18.47	20.00	23.51
5	7.29	9.24	11.07	11.64	12.37	13.39	15.09	16.75	20.52	22.11	25.74

1. (3 points) State null and alternative hypotheses.

   Solution: H_0: treatment is independent of result
   H_1: treatment is not independent of result

   Using parameters, if $\theta_{j\,|\,i} = \mathsf{P}\,(Y = j \,|\, X = i)$, then $H_0 \colon \theta_{j\,|\,i} = \theta_j$ for all i, and $H_1 \colon \theta_{j\,|\,1} \neq \theta_{j\,|\,2}$ for some j.

2. (7 points) Find numerical values for the expected counts in each cell in the table.

   Solution: The expected counts are (row sum)(column sum)/(table sum).

	No germination	Growth/no seed production	Produces seed
Control	8	11	81
Treatment	8	11	81

3. (7 points) Find the numerical value of the test statistic.

   Solution:

$$X^2 = \frac{(11-8)^2}{8} + \frac{(5-8)^2}{8} + \frac{(17-11)^2}{11} + \frac{(5-11)^2}{11} + \frac{(72-81)^2}{81} + \frac{(90-81)^2}{81} = 10.8$$

4. (2 points) Circle choices or fill in the blanks.

   Solution: The p-value for this hypothesis test is equal to the area to the right of 10.8 below the density of a chi-square distribution with 2 degrees of freedom.

5. (1 points) Find a numerical range for the p-value (for example, $0.01 < p < 0.02$) using the table above.

   Solution: The quantile is between 0.995 and 0.999 so, $0.001 < p < 0.005$.