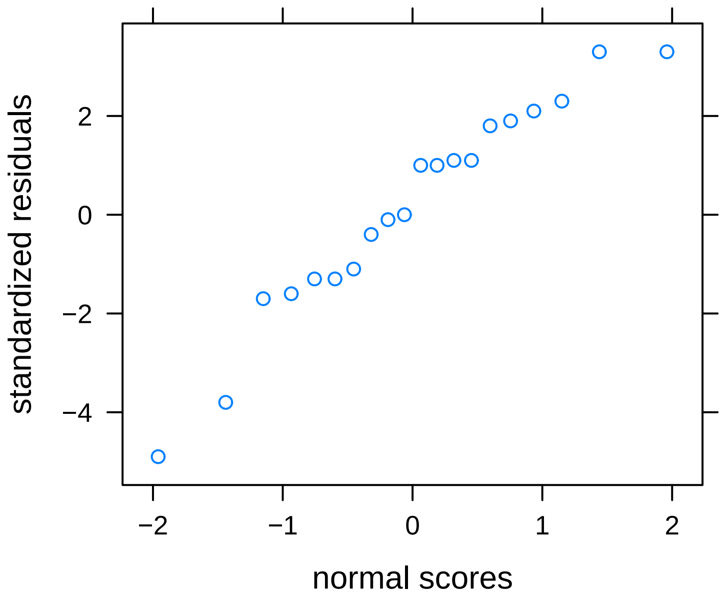
STAT310 - HWK Solution 11

9.1.1 The observed discrepancy statistic is given by $D(r) = \frac{1}{\sigma_0^2}\Sigma_{i=1}^{n}(x_i - \bar{x})^2 = \frac{19}{4}4.79187 = 22.761$. Now $D(R) \sim \chi^2(19)$ distribution, so the P-value is then given by $P(D(R) > 22.761) = 0.248$, which doesn't suggest evidence against the model.



9.1.7

9.1.4 Let $P(A = 0) = \alpha_1, \quad P(A = 1) = \alpha_2, \quad P(B = 0) = \beta_1, \quad P(B = 1) = \beta_2$
Then $(X_{11}, X_{12}, X_{21}, X_{22}) \sim Multinomial(n, \alpha_1\beta_1, \alpha_1\beta_2, \alpha_2\beta_1, \alpha_2\beta_2)$, where $X_{ij} is the counts of cell_{ij}$
Then $P(X_{11} = k|X_{1\bullet}, X_{\bullet 1}) = \binom{x_{1\bullet}}{k}\binom{n-x_{1\bullet}}{x_{\bullet 1}-k}/\binom{n}{x_{\bullet 1}} \sim Hypergeometric(n, x_{\bullet 1}, x_{1\bullet})$, where $x_{1\bullet}$ is the total counts of 1st row, and $x_{\bullet 1}$ is the total counts of the 1st column.
P-value is the probability of getting a value as far or father, out in the tails, than $X_{11} = 2$.
$P - value = \binom{3}{2}\binom{7}{3}/\binom{10}{5} + \binom{3}{3}\binom{7}{2}/\binom{10}{5} = 0.5$. Thus we fail to reject the null hypothesis that gender and political orientation are independent.

9.1.5 By grouping the data into five equal intervals each having length 0.2, the expected counts for each interval are $np_i = 4$, and the observed counts are given in the following table.

Interval	Count
$(0.0, 0.2]$	4
$(0.2, 0.4]$	7
$(0.4, 0.6]$	3
$(0.6, 0.8]$	4
$(0.8, 1.0]$	2

The Chi-squared statistic is equal to 3.50 and the P-value is given by $P(X^2 \geqslant 3.5) = 0.4779$, here $X^2 \sim \chi^2(4)$. Therefore, we have no evidence against the Uniform model being correct.

9.1.6 First note that if the die is fair, then the expected number of counts for each possible outcome is 1000/6. The chi-squared statistic is thus equal to 9.572 and the P-value is given by $P(X^2 > 9.572) = 0.08831 > 0.05$, here $X^2 \sim \chi^2(5)$. The P-value is smaller than 0.1, so there is evidence that the die might not be fair. However, if we adopt the significance level 0.05, there is still not enough evidence to reject the null hypothesis.

The standardized residuals are:
$r_1 = -0.3111$, $r_2 = 0.9617$, $r_3 = -2.0939$, $r_4 = -1.4142$, $r_5 = 1.3859$, $r_6 = 1.4708$