STAT310 - HWK Solution 8

(a)
```
> s=c(24.81, 23.52, 23.58, 23.08, 19.99, 28.98, 24.08, 21.75, 19.23, 23.30)
> mean(s)
[1] 23.232
> sd(s)
[1] 2.69162
```

(b)
```
> t.test(s, conf.level = 0.90)

        One Sample t-test

data:  s     t = 27.2944,   df = 9,   p-value = 5.766e-10
alternative hypothesis: true mean is not equal to 0
90 percent confidence interval:
 21.67172 24.79228
sample estimates: mean of x
   23.232
```

Thus, the 90% confidence interval using the t distribution is (21.67172, 24.79228)

(c)
```
> library(boot)
> get.mean = function(x,indices) { return(mean(x[indices])) }
> boot(s, get.mean, R=10000)

ORDINARY NONPARAMETRIC BOOTSTRAP

Call: boot(data = s, statistic = get.mean, R = 10000)

Bootstrap Statistics :
    original     bias    std. error
t1*   23.232 -0.0024803   0.8052411

> 23.232-1.645*0.8052411
[1] 21.90738
> 23.232+1.645*0.8052411
[1] 24.55662
```

Thus the 90% confidence interval using the non-parametric bootstrap is (21.90738, 24.55662)

(d) Based on Example 7.1.2 of the text book, if $s = (x_1, x_2......x_n), x_i \sim i.i.d.N(\mu, \sigma_0^2)$, and the prior distribution of μ is $\mu \sim N(\mu_0, \tau_0^2)$, then $\mu|s \sim N((\frac{1}{\tau_0^2} + \frac{n}{\sigma_0^2})^{-1}(\frac{\mu_0}{\tau_0^2} + \frac{n}{\sigma_0^2}\bar{x}), (\frac{1}{\tau_0^2} + \frac{n}{\sigma_0^2})^{-1})$.
In this problem $\sigma_0 = 4, \mu_0 = 20, \tau_0 = 2, n = 10, \bar{x} = 23.232$, then posterior distribution of μ is $\mu|s \sim N(22.309, 1.143)$

```
> qnorm(c(0.05, 0.95), 22.309, sqrt(1.143))
```

```
[1] 20.55047 24.06753
```

Thus a 90% credible region is (22.55, 24.07)

(e) Based on Example 7.1.4 of the text book, $s = (x_1, x_2......x_n), x_i \sim i.i.d.N(\mu, \sigma^2)$, let $\nu = \frac{1}{\sigma^2}$, the prior distribution of ν is $Gamma(\alpha_0, \beta_0)$, the prior distribution of $\mu|\nu$ is $N(\mu_0, \frac{\tau_0^2}{\nu})$, then the posterior distributions of $\mu|\nu$ and ν are

$$\mu|\nu, s \sim N((n + \frac{1}{\tau_0^2})^{-1}(\frac{\mu_0}{\tau_0^2} + n\bar{x}), \frac{1}{(n + \frac{1}{\tau_0^2})\nu})$$

$$\nu|s \sim Gamma(\alpha_0 + \frac{n}{2}, \beta_x)$$

$$\beta_x = \beta_0 + \frac{n\bar{x}^2}{2} + \frac{\mu_0^2}{2\tau_0^2} + \frac{n-1}{2}s^2 - \frac{1}{2}(n + \frac{1}{\tau_0^2})^{-1}(\frac{\mu_0}{\tau_0^2} + n\bar{x}^2)^2$$

In this problem, $\alpha_0 = 4, \beta_0 = 64, \mu_0 = 20, \tau_0^2 = 0.2, n = 10, \bar{x} = 23.232$, thus we have

$$\mu|\nu, s \sim N(22.155, 0.0667\nu^{-1})$$

$$\nu|s \sim Gamma(9, 114.0114)$$

(f)
```
> nu.post = rgamma(10000, 9, 114.0114)
> sigma2.post = 1/nu.post
> mu.post = rnorm(10000, 22.155, sqrt(0.0667*sigma2.post))
>
> print(round(quantile(mu.post, c(0.05, 0.95)),1))
  5%  95%
 20.6 23.8
> print(round(quantile(mu.post, c(0.025, 0.975)),1))
 2.5% 97.5%
 20.2  24.1
```