Assignment #10 — Due Monday, April 13, 2009, by 5:00 P.M.

Turn in homework in lecture, discussion, or your TA's mailbox. Indicate the discussion section in which you expect to attend to pick up this assignment on the assignment.

311: Monday 1:20–2:10 **312:** Monday 12:05–12:55

This assignment involves sample questions from Chapter 8.

1. **(Rao-Blackwell Theorem)** Let $X_1, \ldots, X_n$ be an independent sample from a *Bernoulli*(θ) distribution, so that $\mathsf{P}(X_i = 1) = \theta$ and $\mathsf{P}(X_i = 0) = 1 - \theta$.
   (a) Find a minimal sufficient statistic U for θ.
   (b) Find the maximum likelihood estimator $\hat{\theta}$ for θ.
   (c) Suppose a statistician wanted to use $T(s) = (X_1 + X_2)/2$ for an estimator. Using the sufficient statistic U you found in part (a), find the Rao-Blackwell estimator $T_U = \mathsf{E}(T \mid U)$.
   (d) Compute the exact MSE for T and for T_U and verify (assuming $n > 2$) the claim of the Rao-Blackwell theorem in this example.
2. **(Cramer-Rao Lower Bound)** Let $X_1, \ldots, X_n$ be an independent sample from a Geometric$(1/\theta)$ distribution with probability function $p(x) = (1/\theta)(1 - 1/\theta)^x$ for $x = 0, 1, 2, \ldots$ where $\theta > 1$.
   (a) Find a minimal sufficient statistic for θ.
   (b) Find the maximum likelihood estimator $\hat{\theta}$.
   (c) Find the Fisher information $I(\theta)$ for one observation and $nI(\theta)$ for a sample.
   (d) Is the MLE an unbiased estimator?
   (e) What does the Cramer-Rao Lower Bound imply about variance of the MLE?
3. **(Likelihood Ratio Tests)**

   We observe 250 random variables, each of which takes on a value from $0, 1, 2, 3, 4$. The table of observations is

x	0	1	2	3	4
count	103	73	45	25	4

   so 103 of the 250 random variables were observed to be zero, and so on. Consider these two models: (1) $X_i \sim$ Binomial$(4, \theta)$; (2) $X_i \sim$ Multinomial(5).

   Find the MLE for each model and conduct a likelihood ratio test for the binomial model versus the multinomial model. *State hypotheses, calculate the value of the test statistic, compare the value of this statistic to a reference distribution, and compute a p-value.*

Work to do, but not turn in.

- Read Chapter 9.