

Assignment #7 — Due Wednesday, March 11, 2009, by 5:00 P.M.

Turn in homework in lecture, discussion, or your TA's mailbox. Indicate the discussion section in which you expect to attend to pick up this assignment on the assignment.

311: Monday 1:20–2:10**312:** Monday 12:05–12:55

The first several problems are review of important ideas from Chapter 6. The last few problems are about Bayesian inference.

1. Do exercises 6.3.17 and 6.3.18.
2. Several parts of this problem are excellent practice for the make-up exam on Wednesday next week. Assume that the following sample is from a distribution with density

$$f_{\theta}(x) = \theta(1+x)^{-\theta-1}, \quad x > 0$$

where $\theta \in \Omega = (0, \infty)$.

2.2 1.7 1.0 0.7 0.3 3.7 2.9 1.6 0.4 2.6 6.2 24.7 23.2 1.0 2.7

- (a) Find the cumulative distribution function of this distribution.
 - (b) Verify that the density integrates to one and that it is a proper density function.
 - (c) Find an expression for the maximum likelihood estimate of θ and find its numerical estimate.
 - (d) Find a minimal sufficient statistic for this model.
 - (e) Find an expression for the Fisher information for this model. Compute both the observed and plug-in estimates of the Fisher information.
 - (f) Construct an approximate 95% confidence interval for θ based on the fact that for sufficiently large samples that $\hat{\theta} \approx N(0, (nI(\hat{\theta}))^{-1})$.
 - (g) Explain how one might use simulation to estimate the bias of $\hat{\theta}$.
3. For the exponential distribution, verify that equations (6.5.2), (6.5.3), and (6.5.4) hold. (These were assumptions (1)–(3) from lecture notes.)
 4. Do Exercise 7.1.1.
 5. Do Exercise 7.1.4.
 6. Assume that (11, 11, 18, 15, 12, 14, 8, 18) is the realization of a sample from a Poisson(θ) distribution and that we assume that $\pi(\theta)$ is the Gamma(2, 0.2) density. Use `ggamma()` from the `prob.R` file to graph the prior and posterior densities of θ . Find the 0.025 and 0.975 quantiles of the posterior distribution to compute a 95% credible region for θ .

Work to do, but not turn in.

- Read sections 7.1–7.4.