1: (a) 
\[ E(X^k) = \frac{1}{B(\alpha, \beta)} \int_0^1 x^{k+\alpha-1}(1-x)^{\beta-1} = \frac{B(\alpha+k, \beta)}{B(\alpha, \beta)}. \]
(b) 
\[ \text{Var}(X) = E(X^2) - (E(X))^2 = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}. \]

2:
\[ \sum_{k=0}^{3} \frac{c}{(k+1)(k+2)} = c \sum_{k=0}^{3} \left( \frac{1}{k+1} - \frac{1}{k+2} \right) = c(1 - 1/6) = 1 \]
So \( c = 5/4 = 1.25 \)

\[ \begin{array}{c|cccc}
 k & 0 & 1 & 2 & 3 \\
p_k & 5/8 & 5/24 & 10/48 & 15/80 \\
\end{array} \]
So \( EX = 29/48 \), \( EX^2 = 19/16 \). So \( \text{Var}(X) = EX^2 - (EX)^2 = .822 \)
\[ SD(X) = \sqrt{\text{Var}(X)} = .907 \]

3: (a) \( E(Y) = \int_{-\infty}^{\infty} ye^{-|y|/2}dy = 0 \) (by symmetric)
(b) 
\[ EY^2 = \int_{-\infty}^{\infty} y^2e^{-|y|/2}dy = \int_0^{\infty} y^2e^{-y}dy = \Gamma(3) = 2 \]
So \( \text{Var}(Y) = E(Y^2) - (EY)^2 = 2 \) and \( SD(Y) = \sqrt{2} \)

4: (a)(Method 1) \( R \) is HyperGeometric(M=2,N=10,n=5). So \( EX = nM/N = 1 \)
\[ \text{Var}(X) = n \frac{M}{N} (1 - \frac{M}{N}) \frac{N-n}{N-1} = 5 \frac{2}{10} \frac{8}{10} \frac{5}{9} = 4/9 \]

( Method 2 ) 
\[ R = \sum_{i=1}^{5} I_{(ith=red)}. \]
\[ EX = \sum_{i=1}^{5} P(ith = \text{green}) = 5 \frac{2}{10} = 1 \]
\[ EX^2 = \sum_{i=1}^{5} P(ith = \text{green}) + 2 \sum_{i<j} P(ith = \text{green}, jth = \text{green}) \]
\[ P(ith = \text{green}, jth = \text{green}) = P(1st = \text{green}, 2nd = \text{green}) = (2/10)(1/9) = 1/45 \]
So \( EX^2 = 1 + 4/9 = 13/9 \)
\[ \text{Var}(X) = EX^2 - (EX)^2 = 13/9 - 1 = 4/9 \]
(b) Similarly, \( E(G) = \frac{3}{2} \) and \( \text{Var}(G) = \frac{7}{12} \).

(c) \( \text{Cov}(R, G) = \frac{1}{2}(\text{Var}(R + G) - \text{Var}(R) - \text{Var}(G)) = \frac{1}{2}\left(\frac{25}{36} - \frac{4}{9} - \frac{7}{12}\right) = -\frac{1}{6} \)

\[
\text{Corr}(R, G) = \frac{\text{Cov}(R, G)}{\sqrt{\text{Var}(R)\text{Var}(G)}} = -0.3273
\]

5: \( P(X = k) = \frac{\theta^k}{k!} e^{-\theta}, k = 0, 1, 2, \ldots \)

\[
m(s) = E(e^{sX}) = \sum_{k=0}^{\infty} e^{sk} \frac{\theta^k}{k!} e^{-\theta}
\]

\[
= e^{-\theta} \sum_{k=0}^{\infty} \frac{(e^{s})^k}{k!} = e^{-\theta} e^{e^s}
\]

\[
m'(s) = \theta e^{(e^s-1)} e^s
\]

\[
EX = m'(0) = \theta
\]

\[
m''(s) = m'(s)(\theta e^s)^2 + m(s)e^s
\]

\[
EX^2 = m''(0) = \theta^2 + \theta
\]

\[
m'''(s) = m''(s)(\theta e^s)^2 + m'(s)(\theta e^s)\theta e^s + m'(s)e^s + m(s)e^s
\]

\[
EX^3 = m'''(0) = \theta^3 + 3\theta^2 + \theta
\]

6: \[
m_Y(s) = Ee^{sY} = Ee^{s(aX+b)}
= e^{bs} Ee^{asX} = e^{bs} m_X(as)
\]

\[
c_Y(s) = Ee^{isY} = Ee^{is(aX+b)}
= e^{ibs} Ee^{iasX} = e^{ibs} c_X(as)
\]

7: From 7, we have \( c_{X/2}(s) = c_X(s/2) = e^{-\frac{|s|}{2}} \). Similarly \( c_{Y/2}(s) = e^{-\frac{|s|}{2}} \). X and Y are independent

\[
c_Z(s) = c_{X/2}(s)c_{Y/2}(s) = e^{-\frac{|s|}{2}} e^{-\frac{|s|}{2}} = e^{-|s|}
\]

So Z is Cauchy distribution.

8: \( P(-1 \leq X \leq 1) \) and \( P(-1 \leq Z \leq 1) \) are approximated by R very similarly, which are both around 0.5.

> `pcauchy(1) - pcauchy(-1)` is returned as value 0.5 in R.