1: (a)$E(X^k) = \frac{1}{B(\alpha,\beta)} \int_0^1 x^{k+\alpha-1}(1-x)^{\beta-1} = \frac{B(\alpha+k,\beta)}{B(\alpha,\beta)}$.
(b)$Var(X) = E(X^2) - (E(X))^2 = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$.

2:

$$\sum_{k=0}^{3} \frac{c}{(k+1)(k+2)} = c\sum_{k=0}^{3}(\frac{1}{k+1} - \frac{1}{k+2}) = c(1-1/6) = 1$$

So c=5/4=1.25

k	0	1	2	3
p_k	5/8	5/24	10/48	15/80

So $EX = 29/48$, $EX^2 = 19/16$. So $Var(X) = EX^2 - (EX)^2 = .822$

$$SD(X) = \sqrt{VarX} = .907$$

3: (a) $E(Y) = \int_{-\infty}^{\infty} ye^{-|y|}/2dy = 0$ (by symmetric)
(b)

$$EY^2 = \int_{-\infty}^{\infty} y^2 e^{-|y|}/2dy = \int_0^{\infty} y^2 e^{-y}dy = \Gamma(3) = 2$$

So $Var(Y) = E(Y^2) - (EY)^2 = 2$ and $SD(Y) = \sqrt{2}$

4: (a)(Method 1) R is HyperGeometric(M=2,N=10,n=5). So $EX = nM/N = 1$

$$Var(X) = n\frac{M}{N}(1 - \frac{M}{N})\frac{N-n}{N-1} = 5 * \frac{2}{10} * \frac{8}{10} * \frac{5}{9} = 4/9$$

(Method 2) $R = \sum_{i=1}^{5} I_{\{ith=red\}}$.

$$EX = \sum_{i=1}^{5} P(ith = green) = 5 * 2/10 = 1$$

$$EX^2 = \sum_{i=1}^{5} P(ith = green) + 2\sum_{i<j} P(ith = green, jth = green)$$

$$P(ith = green, jth = green) = P(1st = green, 2nd = green) = (2/10)(1/9) = 1/45$$

So $EX^2 = 1 + 4/9 = 13/9$

$$Var(X) = EX^2 - (EX)^2 = 13/9 - 1 = 4/9$$

(b)Similarly, $E(G) = 3/2$ and $Var(G) = 7/12$.
(c)$Cov(R, G) = \frac{1}{2}(Var(R+G) - Var(R) - Var(G)) = \frac{1}{2}(\frac{25}{36} - \frac{4}{9} - \frac{7}{12}) = -\frac{1}{6}$

$$Corr(R, G) = \frac{Cov(R, G)}{\sqrt{(Var(R)Var(G))}} = -.3273$$

5:$P(X = k) = \frac{\theta^k}{k!}e^{-\theta}, k = 0, 1, 2, \ldots$

$$m(s) = E(e^{sX}) = \sum_{k=0}^{\infty} e^{sk}\frac{\theta^k}{k!}e^{-\theta}$$

$$= e^{-\theta}\sum_{k=0}^{\infty}\frac{(e^s\theta)^k}{k!} = e^{-\theta}e^{e^s\theta}$$

$$m'(s) = e^{\theta(e^s-1)}\theta e^s$$

$$EX = m'(0) = \theta$$

$$m''(s) = m'(s)(\theta e^s)^2 + m(s)e^s$$

$$EX^2 = m''(0) = \theta^2 + \theta$$

$$m'''(s) = m''(s)(\theta e^s)^2 + m'(s)(\theta e^s)\theta e^s + m'(s)e^s + m(s)e^s$$

$$EX^3 = m'''(0) = \theta^3 + 3\theta^2 + \theta$$

6:

$$m_Y(s) = Ee^{sY} = Ee^{s(aX+b)}$$
$$= e^{bs}Ee^{asX} = e^{bs}m_X(as)$$

$$c_Y(s) = Ee^{isY} = Ee^{is(aX+b)}$$
$$= e^{ibs}Ee^{iasX} = e^{ibs}c_X(as)$$

7: From 7, we have

$$c_{X/2}(s) = c_X(s/2) = e^{-\frac{|s|}{2}}$$

Similarly $c_{Y/2}(s) = e^{-\frac{|s|}{2}}$. X and Y are independent

$$c_Z(s) = c_{X/2}(s)c_{Y/2}(s) = e^{-\frac{|s|}{2}}e^{-\frac{|s|}{2}} = e^{-|s|}$$

So Z is Cauchy distribution.

8:$P(-1 \le X \le 1)$ and $P(-1 \le Z \le 1)$ are approximated by R very similarly, which are both around 0.5.
$> pcauchy(1) - pcauchy(-1)$ is returned as value 0.5 in R.