1 Solutions for HW9

1: (a) $E(Z) = 1.08, E(W) = 4.92$
(b) No, it does not matter.
(c) Yes, independence matters here.

2:

$$E(X + 1) = \sum_{k=1}^{\infty} kP(X = k)$$
$$= \sum_{k=1}^{\infty} \sum_{j=0}^{k-1} P(X = k)$$
$$= \sum_{j=0}^{\infty} \sum_{k=j+1}^{\infty} P(X = k)$$
$$= \sum_{j=0}^{\infty} P(X > j)$$

3: $X \sim Geometric(\theta)$, so $P(X > k) = \sum_{j=k+1}^{\infty} (1 - \theta)^j \theta = (1 - \theta)^{k+1}$

$$EX = \sum_{0}^{\infty} (1 - \theta)^{k+1} = \frac{1 - \theta}{\theta}$$

4:

$$E(X + 1) = \sum_{0}^{\infty} (k + 1)(1 - \theta)^k \theta$$
$$= -\theta \sum_{0}^{\infty} \left( \frac{d(1 - \theta)^{k+1}}{d\theta} \right)$$
$$= -\theta \frac{d}{d\theta} \left( \sum_{0}^{\infty} (1 - \theta)^{k+1} \right)$$
$$= -\theta \frac{d}{d\theta} \left( \frac{1}{\theta} - 1 \right) = \frac{1}{\theta}$$
5:

\[ E[(X + 2)(X + 1)] = \sum_{0}^{\infty} (k + 2)(k + 1)(1 - \theta)^k \theta \]

\[ = \theta \sum_{0}^{\infty} \frac{d^2(1 - \theta)^{k+2}}{d\theta^2} \]

\[ = \theta \frac{d^2}{d\theta^2} \sum_{0}^{\infty} (1 - \theta)^{k+2} \]

\[ = \theta \frac{d^2}{d\theta^2} \left[ \frac{(1 - \theta)^2}{\theta} \right] \]

\[ = \theta \frac{d^2}{d\theta^2} \left[ \frac{1}{\theta} - 2 + \theta \right] \]

\[ = \frac{2}{\theta^2} \]

6: Let \( I_i \) is 1 if \( i \) is before \( i-1 \), and 0 otherwise, for \( i \) from 2 to 10. Then \( X = 1 + I_2 + \ldots + I_{10} \). Note that \( E(I_i) = 0.5 \). So \( EX = 1 + 0.5 \times 9 = 5.5 \).

7: (a) Let \( I_i \) is 1 if the \( i \)th jack is in the first five cards, and 0 otherwise. Then \( X = I_1 + I_2 + I_3 + I_4 \). Note that \( E(I_i) = \left( \frac{51}{4} \right) \left( \frac{52}{4} \right) = 5/52 \). So \( EX = 4 \times 5/52 = 5/13 \).

(b) Let \( I_i \) is 1 if the \( i \)th jack is before the first queen, and 0 otherwise. Then \( X = I_1 + I_2 + I_3 + I_4 \). Note that \( E(I_i) = 1/5 \). So \( EX = 4/5 \).

(c) Let \( I_i \) is 1 if the \( i \)th jack is before the second queen and 0 otherwise. Then \( X = I_1 + I_2 + I_3 + I_4 \). Note that the two disjoint events, that make \( I_i \) be 1, are ‘the \( i \)th jack is before the first queen’ and ‘the \( i \)th is after the first queen and before the second queen’, each with a probability 1/5. So \( E(I_i) = 2/5 \) and so \( EX = 4 \times 2/5 = 8/5 \).

(d) Let \( I_i \) is 1 if the \( i \)th jack is before the queen spade, and 0 otherwise. Then \( X = I_1 + I_2 + I_3 + I_4 \). Note that \( E(I_i) = 1/2 \). So \( EX = 2 \).

8: (a) \( 1 - \text{pnorm}(240, 250, 25) = .6554 \)

(b) \( 1 - \text{pnorm}(212, 250, 25) = .9357 \)

(c) \( \text{pnorm}(287, 250, 25) - \text{pnorm}(210, 250, 25) = .8758 \)

(d) \( \text{qnorm}(.8, 250, 25) = 271 \)

(e) \( \text{pnorm}(220, 250, 25) + 1 - \text{pnorm}(280, 250, 25) = .2301 \)

(f) \( \text{qnorm}(.75 + .25/2, 250, 25) \times 25 = 28.76 \)

9: (a) \( 1 - \text{pbinom}(430, 1200, .38) = .9375 \)

(b) \( \text{pbinom}(470, 1200, .38) - \text{pbinom}(429, 1200, .38) = .7488 \)

(c) \( \text{pbinom}(1200 \times .38 - 1, 1200, .38) = .4891 \)

(d) \( \text{dbinom}(1200 \times .38, 1200, .38) = .0237 \)

(e) \( \text{qbinom}(.95, 1200, .38) = 484 \)