1 Solutions for HW7

1: $y = h(x) = 3x^2, y \geq 0. h'(x) = 6x, h^{-1}(y) = \sqrt{y/3}$ so

$$f_Y(y) = \frac{f_X(h^{-1}(y))}{|h'(h^{-1}(y))|} = f_X(\sqrt{y/3})/|h'(\sqrt{y/3})| = \frac{\lambda e^{-\lambda\sqrt{y/3}}}{2\sqrt{3y}}, y \geq 0$$

2: $y = h(x) = 1/x, h'(x) = -1/x^2, h^{-1}(y) = 1/y.$ so

$$f_Y(y) = \frac{f_X(1/y)}{|h'(1/y)|} = \frac{\lambda e^{-\lambda/y}}{y^2}, y > 0$$

3:

$$f_Y(y) = \frac{e^{-\frac{1}{2}y^{\frac{2}{3}}}}{3\sqrt{2\Pi}y^{2/3}}, y > 0$$

4:$y = h(x) = \frac{1}{1+x}, 1/2 \leq y \leq 1, h'(x) = -\frac{1}{(1+x)^2}, h^{-1}(y) = 1/y - 1,$

$$f_Y(y) = \frac{f_X(1/y-1)}{|h'(1/y-1)|} = 1/y^2, 1/2 \leq y \leq 1$$

5: $f_X(x) = \frac{x^{a-1}}{B(a,1)}, x \in (0,1),$
$y = h(x) = -5ln(x), y > 0. h'(x) = -5/x, h^{-1}(y) = e^{-y}$, So

$$f_Y(y) = \frac{f_X(e^{-y})}{|h'(e^{-y})|} = \frac{e^{-ay}}{B(a,1)} = \frac{a}{5}e^{-ay/5}, y > 0$$

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6: $f_X(x) = \frac{\lambda^2 x}{\Gamma(2)}e^{-\lambda x} = \lambda^2 x e^{-\lambda x}, x > 0$
$y = h(x) = \sqrt{x}, h'(x) = \frac{1}{2\sqrt{x}}, h^{-1}(y) = y^2$

$$f_Y(y) = \frac{f_X(y^2)}{|h'(y^2)|} = \frac{\lambda^2 y^2 e^{-\lambda y^2}}{1/(2y)} = 2\lambda^2 y^3 e^{-\lambda y^2}, y > 0$$

11: (a):

	Y=0	Y=1
X=0	c	c
X=1	c	2c
X=2	c	3c

So 9c=1, c=1/9.
(b) P(X=0)=2/9, P(X=1)=3/9=1/3, P(X=2)=4/9
(c) the marginal distribution of Y P(Y=0)=1/3, P(Y=1)=2/3

(d) P(X=Y)=P(X=0, Y=0)+P(X=1, Y=1)=1/9+2/9=1/3