1 Solutions for HW4

1:(a) $P = \binom{120}{25} * .25^{25} * .75^{95} = .051$
(b) $\theta = \frac{1}{64}$
$P = 1 - \binom{40}{0}(\frac{1}{64})^0 * (\frac{63}{64})^{40} - \binom{40}{1}(\frac{1}{64})^1 * (\frac{63}{64})^{39} = .1292.$

2:(a) X: the number of not double ones until someone rolls double ones, then $X \sim G(\theta)$, where $\theta = (\frac{1}{6})(\frac{1}{6}) = \frac{1}{36}$.
$P = \sum_{i=9}^{\infty}(1-\theta)^i\theta = (1-\theta)^9 = (\frac{35}{36})^9 = .7761.$
(b)$P(Awins) = \theta\sum_{i=0}^{\infty}(1-\theta)^{5i} = \frac{\theta}{1-(1-\theta)^5} = .2114$
$P(Bwins) = \theta(1-\theta)\sum_{i=0}^{\infty}(1-\theta)^{5i} = \frac{\theta(1-\theta)}{1-(1-\theta)^5} = .2056$
$P(Cwins) = \theta(1-\theta)^2\sum_{i=0}^{\infty}(1-\theta)^{5i} = \frac{\theta(1-\theta)^2}{1-(1-\theta)^5} = .1998$
$P(Dwins) = \theta(1-\theta)^3\sum_{i=0}^{\infty}(1-\theta)^{5i} = \frac{\theta(1-\theta)^3}{1-(1-\theta)^5} = .1943$
$P(Ewins) = \theta(1-\theta)^4\sum_{i=0}^{\infty}(1-\theta)^{5i} = \frac{\theta(1-\theta)^4}{1-(1-\theta)^5} = .1889$

3:(a) $f(\theta) = P(X_1 = 2)P(X_2 = 3)P(X_3 = 5) = \frac{e^{-3\theta}}{2!3!5!}\theta^{10}$
$f'(\theta) = 0$ implies that $\theta = 10/3$

(b) Plug $\theta = 10/3$ into $f(\theta)$ and the answer is .0053392.

4:(a) $P(Y = 0) = .42$, $P(Y = 1) = .46$, $P(Y = 2) = .12$.
(b) Suppose $Y \sim Binom(2, \theta)$, then three equations hold: $(1-\theta)^2 = .42$,$2\theta(1-\theta) = .46$,$\theta^2 = .12$. However, there is no valid solution. Hence Y does not follow a binomial distribution.

5:(a)$P(Y = 0) = (1-\theta_1)(1-\theta_2)$, $P(Y = 1) = \theta_1(1-\theta_2) + (1-\theta_1)\theta_2$, $P(Y = 2) = \theta_1\theta_2$.
(b)Suppose $Y \sim Binom(2, \theta)$, then three equations hold: $(1-\theta)^2 = (1-\theta_1)(1-\theta_2)$,$2\theta(1-\theta) = \theta_1(1-\theta_2) + (1-\theta_1)\theta_2$,$\theta^2 = \theta_1\theta_2$. The only solution is that $\theta_1 = \theta_2$, which contradicts with the assumption.