1 Solutions for HW4

1: \{2,4,1\} or \{2,4,6\} or \{2,5,1\} or \{2,5,6\} or \{3,4,1\} or \{3,4,6\} or \{3,5,1\} or \{3,5,6\}

2: \[ P(W_1|W_2) = \frac{P(W_2|W_1)P(W_1)}{P(W_2|W_1)P(W_1)+P(W_2|R_1)P(R_1)} = \frac{(3/5)(2/3)}{(3/5)(2/3)+(2/5)(1/2)} = 2/3 \]

3: (a) \( P(1) = P(2) = P(3) = .09 \). Suppose \( P(4) = x \) and then \( P(5) = .73 - x \). \( P(A) = .18 + x = P(B) = P(C), P(A \cap B) = P(1,4) = .09 + x = P(A \cap C) = P(B \cap C) \). So A, B, C are pairwise independent. \( P(A \cap B) = P(A)P(B) \) implies \( .18 + x^2 = .09 + x \). So \( x = .72 \), which implies that 72 balls are numbered as 4 and 1 ball is numbered as 5.

(b) \( P(A \cap B \cap C) = .72, P(A)P(B)P(C) = .9^3 = .729 \). So A, B, C are not mutually independent.

4: \( P(2 \leq X \leq 12) = \sum_{x=2}^{12} (1 - \theta)^x \theta = (1 - \theta)^2 - (1 - \theta)^{13} \)

5: \( \log f = \log(\theta^k) + 6\log \theta + 2\log(1 - \theta), \log f' = 0 \) implies \( \theta = 3/4 \).

6: \( \log f = -\lambda + 6\log \lambda - \log(6!), \log f' = 0 \) implies \( \lambda = 6 \).

7: (a) \( X \sim Binom(6, 0.2), P(X = 2) = .246 \)
(b) \( X \sim Hypergeometric, N = 10, M = 2, n = 6, P = \binom{2}{3} \binom{8}{8} / \binom{10}{6} = 1/3 \)
(c) \( X \sim NB(2, 0.2), P(X = 4) = \binom{5}{4} 0.2^4 0.8^4 = 0.0819 \)
(d) \( X \sim NB(2, 0.8), P(X = 4) = \binom{5}{4} 0.8^4 0.2^4 = 0.00512 \)
(e) \( X \sim G(2), P(X \geq 2) = 1 - P(X = 0) - P(X = 1) = .64 \)
(f) \( P(1) = P(\text{one blue ball before the first red ball}) - P(\text{no blue ball before the first red ball}) = 1 - (8/10)(2/9) - (2/10) = .622 \)

8: Yes, \( X = I_{A \cap B} \).

9: \( Y \) is a random variable if and only if \( X > 1 \).

10: (a) \( P(X = 0) = 1/2, P(X = 2) = 1/4, P(X = 3) = 1/4 \).
(b) \( P(Y = 0) = P(X_1 = 0)P(X_2 = 0)P(X_3 = 0) = 1/8, P(Y = 2) = \binom{3}{2} P(X_1 = 0, X_2 = 0, X_3 = 2) = 3/16, P(Y = 3) = 3/16, P(Y = 4) = 3/32, P(Y = 5) = \binom{3}{3} \binom{7}{2} P(X_1 = 0, X_2 = 2, X_3 = 3) = 3/32, P(Y = 7) = 3/64, P(Y = 8) = 3/64, P(Y = 9) = 1/64 \).
(c) \( P(Z = 8) = 1/64, P(Z = 12) = 3/64, P(Z = 18) = 3/64, P(Z = 27) = 1/64, P(Z = 0) = 7/8 \).

11: \( P(Z = z) = \sum_{k=0}^{z} P(X = k \cap Y = z - k) = \sum_{k=0}^{z} P(X = k)P(Y = z - k) = \sum_{k=0}^{z} \binom{m}{k} \theta^k (1 - \theta)^{m-k} \binom{n}{z-k} \theta^{z-k} (1 - \theta)^{n-z+k} = \sum_{k=0}^{z} \binom{m}{k} \theta^z (1 - \theta)^{m+n-z} = \binom{m+n}{z} \theta^z (1-
\( \theta^{m+n-z} \), so \( Z \sim Binom(m+n, \theta) \)

12: \( P(Z = X + Y = 10) = 1 \), so \( Z \) is not a binomial distributed random variable although \( X \) and \( Y \) are so. The reason is that \( X \) is not independent of \( Y \).