1 Solutions for HW4

1: {2,4,1} or {2,4,6} or {2,5,1} or {2,5,6} or {3,4,1} or{3,4,6} or {3,5,1} or {3,5,6}

2: $P(W_1|W_2) = \frac{P(W_2|W_1)P(W_1)}{P(W_2|W1)P(W_1)+P(W_2|R_1)P(R_1)} = \frac{(3/5)(2/3)}{(3/5)(2/3)+(2/5)(1/2)} = 2/3$

3:(a)$P(1) = P(2) = P(3) = .09$. Suppose$P(4) = x$ and then $P(5) = .73 - x$. $P(A) = .18 + x = P(B) = P(C)$, $P(A \cap B) = P(1,4) = .09 + x = P(A \cap C) = P(B \cap C)$. So A, B, C are pairwise independent. $P(A \cap B) = P(A)P(B)$ implies $(.18 + x)^2 = .09 + x$. So $x = .72$, which implies that 72 balls are numbered as 4 and 1 ball is numbered as 5.

(b)$P(A \cap B \cap C) = .72$, $P(A)P(B)P(C) = .9^3 = .729$. So A,B,C are not mutually independent.

4: $P(2 \leq X \leq 12) = \sum_{x=2}^{12}(1-\theta)^x\theta = (1-\theta)^2 - (1-\theta)^{13}$

5: $logf = log\binom{8}{6} + 6log\theta + 2log(1-\theta)$, logf'=0 implies $\theta = 3/4$.

6: $logf = -\lambda + 6log\lambda - log(6!)$, logf'=0 implies $\lambda = 6$.

7: (a)$X \sim Binom(6, 0.2)$, $P(X = 2) = .246$
(b)$X \sim Hypergeometirc$, $N = 10$, $M = 2$, $n = 6$, $P = \binom{2}{2}\binom{8}{4}/\binom{10}{6} = 1/3$
(c)$X \sim NB(2, 0.2)$, $P(X = 4) = \binom{5}{1}0.2^2 0.8^4 = .0819$
(d)$X \sim NB(2, 0.8)$, $P(X = 4) = \binom{5}{1}0.8^2 0.2^4 = .00512$
(e)$X \sim G(.2)$, $P(X \geq 2) = 1 - P(X = 0) - P(X = 1) = .64$
(f)P=1-P(one blue ball before the first red ball)-P(no blue ball before the first red ball)=1-(8/10)(2/9)-(2/10)=.622

8: Yes, $X = I_{A\cap B}$.

9: Y is a random variable if and only if $X > 1$.

10: (a) $P(X = 0) = 1/2$,$P(X = 2) = 1/4$,$P(X = 3) = 1/4$.
(b) $P(Y = 0) = P(X_1 = 0)P(X_2 = 0)P(X_3 = 0) = 1/8$,$P(Y = 2) = \binom{3}{1}P(X_1 = 0, X_2 = 0, X_3 = 2) = 3/16$, $P(Y = 3) = 3/16$,$P(Y = 4) = 3/32$,$P(Y = 5) = \binom{3}{1}\binom{2}{1}P(X_1 = 0, X_2 = 2, X_3 = 3) = 3/16$,$P(Y = 6) = 3/32$,$P(Y = 7) = 3/64$,$P(Y = 8) = 3/64$,$P(Y = 9) = 1/64$.
(c)$P(Z = 8) = 1/64$,$P(Z = 12) = 3/64$,$P(Z = 18) = 3/64$,$P(Z = 27) = 1/64$,$P(Z = 0) = 7/8$.

11: $P(Z = z) = \sum_{k=0}^{z} P(X = k \cap Y = z - k) = \sum_{k=0}^{z} P(X = k)P(Y = z - k) = \sum_{k=0}^{z}\binom{m}{k}\theta^k(1-\theta)^{m-k}\binom{n}{z-k}\theta^{z-k}(1-\theta)^{n-z+k} = \sum_{k=0}^{z}\binom{m}{k}\binom{n}{z-k}\theta^z(1-\theta)^{m+n-z} = \binom{m+n}{z}\theta^z(1-$

$\theta)^{m+n-z}$, so $Z \sim Binom(m+n, \theta)$

12: $P(Z = X+Y = 10) = 1$, so Z is not a binomial distributed random variable although X and Y are so. The reason is that X is not independent of Y.