1 Solutions for HW3

1: (a)Define $A_n(x)$= number of sequences of coin tosses length n where the longest fun of heads is x or less
(b)$P = \frac{A_{15}(5)-A_{15}(4)}{2^{15}} = .0972$

2: (a)P(two or more heads)=1−P(no heads)-P(one head)= $1 - 0.5^4 + \binom{4}{1}0.5^4 = 0.6875$
(b)P(two or more heads|the first is head)=1-P(no head if rolling three times)= $1-0.5^3 = .875$
(c)P(two or more heads|at least one head)=P(two or more heads)/P(one or more heads) $= \frac{.6875}{1-0.5^4} = .7333$

3: By Bayes rule

$$P(A|8red) = \frac{P(A, 8red)}{P(8red)} = \frac{P(8red|A)P(A)}{P(8red|A)P(A) + P(8red|B)P(B)}$$

$$= \frac{1 * 1/2}{1 * \frac{1}{2} + (\frac{1}{2})^8 * \frac{1}{2}} = .9961$$

4: (a) P(two or more dice are 1s)=1-P(no 1)-P(only 1 dice is 1)=$1 - (\frac{5}{6})^5 - \binom{5}{1}(\frac{1}{6})(\frac{5}{6})^4 =$ .1962
(b) P(two or more 1s |1st=1)=P(one or more 1s in another four dice)=$1 - (\frac{5}{6})^4 = .5177$
(c)P(two or more 1s|1st=2)=0.1319
(d) P=$\frac{P(twoormore1s)}{P(atleastone1)} = \frac{.1962}{1-(\frac{5}{6})^5} = .3280$

5: (a)7 stars, 2 bars, P=.00703
(b) P(1st=2)=.125
(c) P(sum=10|1st=2)=P(sum=8, rolling 2 times)=$\frac{7}{64}$
(d) P(1st=2|sum=10)=.1945

6:(a): P(sum=4|sum=4 or 7)=$\frac{P(sum=4)}{P(sum=4)+P(sum=7)} = \frac{3/36}{(3/36)+(6/36)} = 3/9 = 1/3$
(b) $p_2 = p_3 = p_{12} = 0$ and $p_7 = p_1 1 = 1$, and $p_4 = 1/3$ For other i. $P(sum = i|sum = ior7) = \frac{P(sum=i)}{P(sum=i)+P(sum=7)}$. $p_5 = 4/10 = 2/5, p_6 = 5/11, p_8 = 5/11, p_9 = 4/10, p_{10} = 3/9$
(c) By the law of total probability

$$P(win) = \sum_{i=1}^{12} P(win|first = i)P(first = i) = (3/36)*(3/9)+(4/36)*(4/10)+\cdots+(2/36)*1$$

$$= 244/495 = 0.4929$$

7:P(A) = P(B) = P(C) = P(E) = 1/2, P(D) = 3/4.
P(A and B) = 0, P(A and C) = 1/4, P(A and D) = 3/8, P(A and E) = 1/4, P(B and C)

= 1/4, P(B and D) = 3/8, P(B and E) = 1/4, P(C and D) = 1/4, P(C and E) = 1/4, P(D and E) = 1/4,

(a) A and B are disjoint.
(b) Independent pairs are: AC, AD, AE, BC, BD, BE, CE
(c) Yes. P(ACE)=P(2)=1/8=P(A)P(C)P(E), and AC, CE, AE are independent. So they are mutually indpendent. We also can check BCE are mutually independent.

8:Since $P(A \cap B) = P(A)$, $P(B|A) = \frac{P(A \cap B)}{P(A)} = 1$

9: $P(A \cap B) = P(A) > P(A)P(B)$, since $P(B) < 1$, so A and B are not independent.