

1 Solutions for HW2

1:

$$\begin{aligned}
 P(A \cup B \cup C) &= P(A \cup B) + P(C) - P((A \cup B) \cap C) \\
 &= P(A) + P(B) - P(A \cap B) + P(C) - P((A \cap C) \cup (B \cap C)) \\
 &= P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) \\
 &\quad - P(B \cap C) + P(A \cap B \cap C)
 \end{aligned}$$

2:(a) $|S| = 6^{20}$, $|A| = \binom{20}{2} 5^{18} P = \frac{|A|}{|S|} = .198$

(b) $P(\text{at least one } 6) = 1 - P(\text{no } 6) = 1 - 5^{20} / 6^{20} = .97$

(c) 3 stars, and 19 bars, $|A| = \binom{22}{3}$, $P = \frac{|A|}{|S|} = 4.21 * 10^{-13}$

(d) $P(\max \leq i) = \frac{i^{12}}{6^{12}}$. So $P(\max=5) = P(\max \leq 5) - P(\max \leq 4) = \frac{5^{20} - 4^{20}}{6^{20}} = .0258$

3:(a) $\frac{\binom{48}{1}}{\binom{52}{5}} = 1.84 * 10^{-5}$

(b) $\frac{\binom{13}{1} \binom{48}{1}}{\binom{52}{5}}$

(c) $\frac{\binom{13}{2} \binom{2}{1} \binom{4}{3} \binom{4}{2}}{\binom{52}{5}}$

(d) $\frac{\binom{13}{2} \binom{4}{2} \binom{4}{2} \binom{44}{1}}{\binom{52}{5}}$

(e) $4 * \frac{\binom{13}{5}}{\binom{52}{5}} = 1.98 * 10^{-3}$

4: (a) 6 stars and 12 bars $|A| = \binom{18}{6} = 18564$

(b)

- 5 stars and 7 bars. $|A| = \binom{12}{5}$ Prob = $\frac{\binom{12}{5}}{\binom{18}{6}} = .0427$

- 5 stars and 6 bars. Prob = $\frac{\binom{11}{5}}{\binom{18}{6}} = .0249$

- $|A| = \binom{7}{2} + \binom{7}{1}$, Prob = .00151

5: $|S| = 6^{12}$, $|A| = \frac{12!}{2^6} = .0034$