1 1.2.1

(a): P({1,2})=P({1})+P({2})=1/2+1/3=5/6
(b): P({1,2,3})=P({S})=1
(c): P(A)=1/2 if and only if $A = \{2\}$ or $A = \{1, 3\}$

2 1.2.4

$$P(\{2, 3\}) = P(\{2\}) + P(\{3\}) = 0.5 + 0.3 = 0.8 \neq 0.7$$

So the probability is not valid

3 1.2.9

No! impossible. If we assume P({s})=0 for every s in S, then

$$P(S) = P(\cup_{s\in S}\{s\}) = \sum_{s\in S} P(\{s\}) = 0$$

Note, the second equation is because S is finite or countable. However $P(S) = 1$. This is a contradiction. The assumption P({s})=0 is not true.

4 1.2.10

Yes! For example, Uniform [0,1] distribution. P({s})=0 for every s in [0,1].

5

Since each elementary outcome is assigned to equal probability, $P(A) = 3/5$ if and only if A contains three different elementary outcomes. So the number is $\binom{5}{3} = 10$.

6

By the inclusion-exclusion principle. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. So $P(A \cap B) = 2/3 + 1/4 - 4/5 = 7/60$. We have

$$P(A \cap B^c) = P(A) - P(A \cap B) = 2/3 - 7/60 = 11/20$$

$$P(A^c \cap B) = P(B) - P(A \cap B) = 1/4 - 7/60 = 2/15$$

7

By the inclusion-exclusion principle.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.25 + 0.1 - P(A \cap B)$$

. Note $0 \leq P(A \cap B) \leq min(P(A), P(B)) = 0.1$. So the smallest value of $P(A \cup B) = 0.25$ when B is a subset of A; The largest value is 0.35, when $P(A \cap B) = 0$

8

Similar to the above, $P(A \cap B) = P(A) + P(B) - P(A \cup B) = 0.8 + 0.9 - P(A \cup B)$. Note $0.9 = max(P(A), P(B)) \leq P(A \cup B) \leq 1$. the smallest is $0.7(P(A \cup B) = 1)$, the largest is 0.9(B is subset of A).