Assignment #14 — Due Friday, December 11, 2009, by 5:00 P.M.

Turn in homework in lecture, discussion, or your TA's mailbox (just inside the main entrance of MSC, 1300 University Avenue) Please circle the discussion section you expect to attend to pick up this assignment.

311: Monday 1:20–2:10 **312:** Monday 12:05–12:55 **313:** Tuesday 8:25–9:15

1. Toss a fair coin repeatedly. Let H_n be the number of heads in the first n coin tosses, T_n be the number of tails in the first n coin tosses, and $X_n = H_n - T_n$ be the difference between these. For convenience, you may denote $X_0 = H_0 = T_0 = 0$. The coin is tossed until either $X_n = 10$ (there have been ten heads more than tails) or $X_n = -5$ (there have been five more tails than heads). What is the probability that $X_n = 10$ before $X_n = -5$?
2. Repeat the previous problem, but assume that each individual coin toss has probability 0.6 of being heads.
3. A gambler makes a series of independent bets where each bet is won with probability 1/3 and lost with probability 2/3. However, if he wins the bet, his fortune increases by 2. If he loses, his fortune decreases by 1. If he begins with an initial fortune of 3, what is the probability that he doubles his fortune to 6 (or more) before losing his entire initial fortune.
4. A gambler will make a series of bets until she either raises her initial stake to \$100 from \$20 or loses everything. The probability of winning a bet is θ. For this problem, calculate each probability for three separate values of θ, $\theta_1 = 0.48$, $\theta_2 = 0.5$, and $\theta_3 = 0.52$. When a bet of size b is won (or lost), the total the gambler holds increases (or decreases) by b.
   (a) Find the probability that she raises her fortune to \$100 before going broke if she makes \$1 bets for each value of θ.
   (b) If the gambler takes a bolder strategy and makes a series of \$2 bets, she can win (or lose) with fewer bets, on average. Find the probability of raising her fortune to \$100 before going broke using this strategy for each θ.
   (c) Repeat the previous calculations if the sizes of the bets are \$5, \$10, or \$20.
   (d) Which of the previous strategies is the best for each value of θ?
   (e) A more advanced strategy would allow bets of different sizes depending on the current fortune. For example, a very bold gambler could bet everything if current fortune $a \leq \$50$, and bet $100 - a$ if $a > 50$. By this strategy, the gambler either loses immediately or increases her fortune whenever her fortune is less than half the target and either wins immediately or decreases her fortune whenever the current fortune is more than half the target.

      By conditioning on the result of the first bet using this very bold strategy, find a series of linear equations for the probability of eventually winning for possible initial fortunes a. (You need not do this for all a between 0 and 100. It suffices to begin at $a = 20$ and follow the paths of what might happen. For example, if $g(a)$ is the probability of eventually winning, then $g(20) = \theta g(40) + (1-\theta)g(0) = \theta g(40)$. Then find an equation for $g(40)$ by conditioning on what happens if she starts with that initial fortune.) Solve the equations to find the probability of eventually winning (given the initial fortune is $a = 20$) using this very bold strategy for each θ.
   (f) Suppose that bets must be in whole dollar increments. Try to find the best possible strategy for each θ. You do not need to prove your result. Is it best to be bold or timid and how does this depend on the value of θ?
   (g) What do you notice about the probability of eventually winning for different strategies when $\theta = 1/2$?

Work to do, but not turn in.

- Prepare for the final exams.