

**Assignment #7 — Due Wednesday, October 21, 2009, by 5:00 P.M.**

Turn in homework in lecture, discussion, or your TA's mailbox (just inside the main entrance of MSC, 1300 University Avenue) Please circle the discussion section you expect to attend to pick up this assignment.

**311:** Monday 1:20–2:10**312:** Monday 12:05–12:55**313:** Tuesday 8:25–9:15

1. If  $X \sim \text{Exponential}(\lambda)$ , find the density of  $Y = 3X^2$ .
2. If  $X \sim \text{Exponential}(\lambda)$ , find the density of  $Y = 1/X$ .
3. If  $X \sim N(0, 1)$ , find the density of  $X^3$ .
4. If  $X \sim \text{Uniform}(0, 1)$ , find the density of  $Y = 1/(1 + X)$ .
5. If  $X \sim \text{Beta}(a, 1)$ , find the density of  $Y = -5 \ln(X)$ .
6. If  $X \sim \text{Gamma}(2, \lambda)$ , find the density of  $Y = \sqrt{X}$ .
7. Suppose that  $X$  and  $Y$  are discrete random variables with joint distribution  $P(X = x, Y = y) = c(2 - xy)$  for  $x = 0, 1, 2$  and  $y = 0, 1$  where  $c > 0$  is a constant.
  - (a) Find  $c$ .
  - (b) Find the marginal distribution of  $X$ .
  - (c) Find the marginal distribution of  $Y$ .
  - (d) Find  $P(X = Y)$ .
8. Suppose that  $X \sim \text{Gamma}(2, 1)$  and  $Y \sim \text{Gamma}(5, 1)$  and that  $X$  and  $Y$  are independent.
  - (a) Find the joint density  $f_{X,Y}$ . Draw a sketch of the portion of  $\mathbb{R}^2$  where this joint density is positive.
  - (b) If  $W = X + Y$  and  $Z = X/(X + Y)$ , draw a sketch of the portion of  $\mathbb{R}^2$  where the joint density  $f_{W,Z}$  is positive.
  - (c) Find the joint density  $f_{W,Z}$ .
  - (d) Find the marginal densities of  $W$  and of  $Z$ . If these are examples of named densities, identify them and the associated parameter values.
  - (e) Are  $W$  and  $Z$  independent? Explain.

**Work to do, but not turn in.**

- Read Chapter 2, remainder.
-