

**Assignment #6 — Due Wednesday, October 14, 2009, by 5:00 P.M.**

Turn in homework in lecture, discussion, or your TA's mailbox (just inside the main entrance of MSC, 1300 University Avenue) Please circle the discussion section you expect to attend to pick up this assignment.

**311:** Monday 1:20–2:10

**312:** Monday 12:05–12:55

**313:** Tuesday 8:25–9:15

1. Let  $U \sim \text{Uniform}(0, 1)$ . Find the following probabilities.

- (a)  $P(U \leq 1)$ .
- (b)  $P(U = 1/2)$ .
- (c)  $P(U^2 < 1/4)$ .
- (d)  $P(|2U - 1| > 1/2)$ .
- (e)  $P(|U - 1/2| > 0.3)$ .

2. Let  $W \sim \text{Exponential}(8)$ . Find the following probabilities.

- (a)  $P(W \geq 2)$ .
- (b)  $P(W \leq 10)$ .
- (c)  $P(1 < W < 5)$ .
- (d)  $P(W^2 < 4)$ .
- (e)  $P(W > a)$  for some  $a > 0$ .

3. The density functions for the following random variables are of the form  $f(x) = c \times g(x)$  for a given range; in other words, the densities are proportional to  $g$ . Find the constants  $c$  so that each density integrates to one, and then find the associated probability.

- (a) The density of  $X_1$  is  $f_1(x) = c_1 x^3$  on  $(0, 1)$  and 0 otherwise. Find  $c_1$  and  $P(X_1 < 1/2)$ .
- (b) The density of  $X_2$  is  $f_2(x) = c_2 x(2 - x)$  on  $(0, 2)$  and 0 otherwise. Find  $c_2$  and  $P(X_2 > 1/2)$ .
- (c) (*The Cauchy Distribution*) The density of  $X_3$  is  $f_3(x) = c_3 \frac{1}{1+x^2}$  on  $(-\infty, \infty)$ . Find  $c_3$  and  $P(X_3 < 1/2)$ . (*Hint: recall the derivative of arctan or use the substitution  $x = \tan \theta$ .*)
- (d) The density of  $X_4$  is  $f_4(x) = c_4(x^3 - 3x^2 + 2x + 1)$  on  $(0, 4)$  and 0 otherwise. Find  $c_4$  and  $P(2 < X_4 < 3)$ .

4. Verify that the correct scaling constant for the standard normal density is  $1/\sqrt{2\pi}$  by showing that

$$\int_{-\infty}^{\infty} e^{-x^2/2} dx = \sqrt{2\pi} .$$

The derivation is very similar to the one I did in class to find the correct normalizing constant for the Beta distribution.

Begin by letting  $C = \int_{-\infty}^{\infty} e^{-x^2/2} dx$  so that

$$\begin{aligned} C^2 &= \left( \int_{-\infty}^{\infty} e^{-x^2/2} dx \right) \left( \int_{-\infty}^{\infty} e^{-y^2/2} dy \right) \\ &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-(x^2+y^2)/2} dx dy \end{aligned}$$

Use the one-to-one change of variable  $x = r \cos \theta$  and  $y = r \sin \theta$  for  $0 \leq r < \infty$  and  $0 \leq \theta < 2\pi$  (and find the Jacobian derivative for this change to find  $dx dy$ ) to evaluate this double integral and complete the problem.

5. An absolutely continuous random variable  $X$  has density  $f(x) = c(2x^2 - x^3)$  for  $0 \leq x \leq 2$ .
- Find the value of  $c$ .
  - Find the cumulative distribution function for  $X$ .
  - Find  $P(0.5 < X < 1.5)$ .
6. Let  $Z \sim N(0, 1)$ . Use software or Table D.2 to find the following:
- $P(Z > 1)$ ;
  - $P(Z > -1.49)$ ;
  - $P(-1.96 < Z < 1.96)$ ;
  - the constant  $z$  (accurate to two decimal places) such that  $P(Z > z) = 0.50$ ;
  - $P(|Z| > 2.33)$ .
  - the constant  $z$  (accurate to two decimal places) such that  $P(|Z| < z) = 0.9$ .
7. Let  $Y \sim N(200, 25^2)$ . Use software or Table D.2 to find the following:
- $P(Y > 205)$ ;
  - $P(Y > 192)$ ;
  - $P(180 < Y < 220)$ ;
  - the constant  $y$  (accurate to two decimal places) such that  $P(Y > y) = 0.20$ ;
  - $P(|Y - 200| > 40)$ .
  - the constant  $y$  (accurate to two decimal places) such that  $P(|Y - 200| < y) = 0.9$ .
8. (*The Cauchy distribution*) If  $X$  is absolutely continuous with density

$$\frac{1}{\pi(1+x^2)}, \quad (-\infty < x < \infty)$$

find the cumulative distribution of  $X$  and compute  $P(X > 1/\sqrt{3})$ .

**Work to do, but not turn in.**

- Read Chapter 2, remainder.
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