

Assignment #3 — Due Wednesday, September 23, 2009, by 5:00 P.M.

Turn in homework in lecture, discussion, or your TA's mailbox (just inside the main entrance of MSC, 1300 University Avenue) Please circle the discussion section you expect to attend to pick up this assignment.

311: Monday 1:20–2:10**312:** Monday 12:05–12:55**313:** Tuesday 8:25–9:15

1. A fair coin is tossed 15 times. What is the probability that the length of the longest run of heads is exactly 5? *Hint: follow the example from lecture using a recursion relation.*
 2. A coin is tossed four times. Assume that the coin tosses are independent and that heads and tails are equally likely.
 - (a) What is the probability that two or more tosses are heads?
 - (b) What is the probability that two or more tosses are heads given that the first coin toss is a head?
 - (c) What is the probability that two or more tosses are heads given that at least one of the coin tosses is a head?
 3. In a genetics experiment involving crossing fruit flies, a male fly is equally likely to be genetic type A or B. If the fly is type A, then all offspring in a cross with a particular female will have red eyes. If the fly is type B, then each offspring in a cross with the same female is equally likely to have red eyes or not. Suppose that in the cross, there are eight offspring, and all have red eyes. Given this additional information, what is the probability that the male fly is genetic type A?
 4. Five fair regular six-sided dice are rolled.
 - (a) What is the probability that two or more dice are 1s?
 - (b) What is the probability that two or more dice are 1s given that the first die is a 1?
 - (c) What is the probability that two or more dice are 1s given that the first die is a 2?
 - (d) What is the probability that two or more dice are 1s given that there is at least one 1?
 5. Three fair regular eight-sided dice (with sides numbered 1–8) are rolled.
 - (a) What is the probability that the sum of the die rolls is 10?
 - (b) What is the probability that the first die is a 2?
 - (c) What is the probability that the sum of the die rolls is 10 given that the first die roll is a 2?
 - (d) What is the probability that the first die roll is a 2 given that the sum is 10?
 6. Find the probability of winning the game of craps: do Exercise 1.5.15.
 7. A fair 8-sided die numbered 1–8 is rolled once. Consider these events: $A = \{\text{the roll is odd}\}$, $B = \{\text{the roll is even}\}$, $C = \{\text{the roll is more than 4}\}$, $D = \{\text{the roll is 6 or less}\}$, and $E = \{\text{the roll is between 3 and 6 inclusive}\}$.
 - (a) Find the probability of each event.
 - (b) Find all pairs of events that are disjoint.
 - (c) Find all pairs of these events that are pair-wise independent.
 - (d) Are there three events that are mutually independent? Explain.
 8. Prove that if $A \subset B$ and $0 < P(A)$, then $P(B|A) = 1$.
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9. Prove that if $A \subset B$ and $0 < P(A) \leq P(B) < 1$, then A and B are not independent.

Work to do, but not turn in.

- Read Chapter 2, sections 1–4.
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