Assignment #2 — Due Wednesday, September 16, 2009, by 5:00 P.M.

Turn in homework in lecture, discussion, or your TA's mailbox. Please circle the discussion section you expect to attend to pick up this assignment.

311: Monday 1:20–2:10 **312:** Monday 12:05–12:55 **313:** Tuesday 8:25–9:15

1. Follow the example proof for the inclusion-exclusion theorem

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

and prove a three set version:

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

(The theorem can also be extended to larger unions where the signs alternate and the probabilities of the intersections of all possible sizes from one to the largest are included. You do not need to prove this extension.)

2. Suppose we roll 20 fair six-sided dice.
   (a) What is the probability of rolling exactly two 6's?
   (b) What is the probability of rolling at least one 6?
   (c) What is the probability that the sum of the dice is 23?
   (d) What is the probability that the maximum of the dice is 5?

3. In a well-shuffled ordinary 52-card deck, what is the probability that in a hand of five cards:
   (a) we get four queens (and another card)?
   (b) we get four cards of the same rank (and another card)?
   (c) we get a full house (three cards of one rank and two cards of another rank)?
   (d) we get two pairs (two cards of one rank, two cards of another rank, and another card of a different rank)?
   (e) we get a flush (all cards the same suit)?

4. Consider the equation $x_1 + x_2 + \cdots + x_7 = 12$ and solutions where all of the x_i are non-negative integers.
   (a) How many solutions are there?
   (b) If a solution is selected uniformly at random, what is the probability that:
      i. $x_1 = 5$?
      ii. there are no zeros in the solution?
      iii. the largest x_i is exactly 2?

5. An urn has six balls, each of a different color. Twelve balls are drawn from the urn, replacing the ball and mixing well after each draw. What is the probability that each color appears exactly twice?

Work to do, but not turn in.

- Read Chapter 1, sections 3–7.