Assignment #12 — Due Wednesday, November 26, 2008, by 5:00 P.M.

Turn in homework in lecture, discussion, or your TA’s mailbox. Indicate the discussion section in which you expect to attend to pick up this assignment on the assignment.

311: Monday 1:20–2:10          312: Monday 12:05–12:55

For the review problems, recall that $E(Y) = E(E(Y | X))$ and that $\text{Var}(Y) = E(\text{Var}(Y | X)) + \text{Var}(E(Y | X))$.

1. (Review) Suppose that $X \sim \text{Poisson}(\lambda)$ and that $Y | X \sim \text{Binomial}(X, \theta)$. What is $E(Y)$? What is $\text{Var}(Y)$?

2. (Review) Suppose that $\Theta \sim \text{Beta}(a, b)$ and that $Y | \Theta \sim \text{Binomial}(n, \Theta)$. What is $E(Y)$? What is $\text{Var}(Y)$?

3. (Review) Suppose that $X \sim \mathcal{N}(\mu, \sigma^2)$ and that $Y | X \sim \mathcal{N}(X, 1)$. What is $E(Y)$? What is $\text{Var}(Y)$?

4. Suppose that $X_1, X_2, \ldots \sim \text{i.i.d. Poisson}(5)$. Prove that for some $n$, $P(X_1 + X_2 + \cdots + X_n < 6n) > 0.999$.

5. A bucket has $n$ balls, one of which is black and the rest are white. A ball is drawn uniformly at random from the bucket, and $X_n = n1\{\text{black ball is drawn}\}$. What is $\lim_{n \to \infty} E(X_n)$? Show that $X_n \xrightarrow{P} 0$.

6. Suppose that $X_n \sim \text{Geometric}(\lambda/(\lambda + n))$ for some $\lambda > 0$ for $n = 1, 2, \ldots$ and let $Y_n = X_n/n$.
   
   (a) Find $P(Y_n = 0)$ and show that $\lim_{n \to \infty} P(Y_n = 0) = 0$. Since $P(Y_n = 0) = P(Y_n = x)$ for all $x \in \mathbb{R}$, this implies that even though $Y_n$ is discrete, the limiting distribution (if it exists) is continuous.
   
   (b) Find $E(Y_n)$.
   
   (c) Find $\text{Var}(Y_n)$ and $\lim_{n \to \infty} \text{Var}(Y_n)$.
   
   (d) Let $Y \sim \text{Exponential}(\lambda)$. Show that $Y_n \overset{D}{\to} Y$. (Hint: You can show this either by directly showing convergence of the cumulative distribution function or by showing convergence of the moment generating function.)

7. Let $X_1, X_2, \ldots \sim \text{i.i.d. Poisson}(5)$.
   
   (a) Use that fact that sums of independent Poisson random variables are Poisson and the `ppois()` function in R to find $P\left(\sum_{i=1}^{800} X_i > 3900\right)$.
   
   (b) Use the central limit theorem to write an approximation to $P\left(\sum_{i=1}^{800} X_i > 3900\right)$ in terms of $\Phi$, the cdf of the standard normal distribution, and use `pnorm()` in R to compute this probability.

Work to do, but not turn in.

• Read Chapter 4, section 4.6 and read Chapter 11, section 11.1.