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<th>Exam A</th>
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- Do not forget to write your name on your solution.
- Begin each exam solution on a separate sheet of paper.
- Clearly indicate which exam you are working on.
- Show sufficient work to make very clear your method of solution.
- When a problem asks you to compute a numerical quantity, include both an expression for the exact value and a decimal representation of the exact value rounded off to three or more significant digits. (If the numerical value is 2/1771, then both 0.00113 and $1.13 \times 10^{-3}$ are good answers, but 0.001 is too inaccurate.)
Exam 1C

New York state offers a lottery game called Quick-Draw. In one version of the game, a player chooses four numbers from 1 to 80. The state then chooses twenty numbers from 1 to 80. The number of matches determines the payout.

<table>
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<tr>
<th>Number of matches</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
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<tbody>
<tr>
<td>Payout on a $1 bet</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>5</td>
<td>55</td>
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1. (6 points) Find both exact expressions and decimal values for the probability of each number of matches.

2. (2 points) Find the probability of a payout of $0 on a single bet.

3. A player wagers $1 for each of ten independent bets.
   (a) (4 points) What is the probability that the total payout is $0?
   (b) (4 points) What is the probability that the total payout is at least $100?
   (c) (4 points) What is the probability that the total payout is exactly $10?

Exam 1D

Five fair standard dice are rolled. Two of the dice are red and three are white. Assume that each die roll is uniformly likely to result in 1, 2, 3, 4, 5, or 6, and that the die rolls are mutually independent.

1. (5 points) What is the probability that two or fewer 1s are rolled?
2. (5 points) What is the probability that the sum of the five dice is eight?
3. (5 points) What is the probability that both red dice are 1s given that the sum of all five dice is eight?
4. (5 points) What is the probability that the maximum of the red dice equals the maximum of the white dice?

Exam 2D

Suppose that $X$ and $Y$ are independent random variables where $X \sim \text{Geometric}(1/3)$ and $Y \sim \text{Poisson}(2)$.

1. (5 points) Find $P(X > 5)$.
2. (5 points) Find $P(Y < 3)$.
3. (5 points) Find $P(X + Y = 3)$.
4. (2 points) Find $P(X = Y = k)$ for $k = 0, 1, 2$.
5. (3 points) Find $P(X = Y)$. 
Exam 2E

A random variable $X$ has density $f(x) = c/(2 + x)^2$ for $x > 0$ for some $c$.

1. (5 points) Find the value of $c$ so that $f$ is a valid density.
2. (4 points) Find the cumulative distribution function for $X$.
3. (4 points) Calculate $P(X > 1)$.
4. (4 points) Find the number $m$ such that $P(X < m) = 0.5$.
5. (3 points) If $U \sim \text{Uniform}(0, 1)$, find a function $g$ so that $g(U)$ has density $f$.

Exam 2F

Random variables $X$ and $Y$ have joint density $f(x, y) = 6xy$ for $x > 0$, $y > 0$, and $x + 2y < 2$.

1. (5 points) Find the marginal density of $X$.
2. (5 points) Find the marginal density of $Y$.
3. (4 points) What is $P(Y < 1/4)$?
4. (4 points) What is the conditional density of $Y$ given $X = 1$?
5. (2 points) What is $P(Y < 1/4 \mid X = 1)$?

Exam 3C

A bucket contains colored balls that are drawn out one at a time uniformly at random. When a white ball is drawn, it is replaced. When a red ball is drawn, it is replaced along with an additional red ball. The bucket initially contains one white and one red ball.

1. (5 points) What is the expected number of white balls drawn before the first red ball is drawn?
2. (5 points) What is the expected number of white balls drawn before the fifth red ball is drawn?
3. (5 points) Let $R$ be the number of red balls drawn before the first white ball is drawn. What is $P(R = k)$ for $k = 0, 1, 2, \ldots$?
4. (5 points) What is $E(1/R)!$?

Exam 3D

Random variable $X \sim \text{Poisson}(1/2)$. If $X = 0$, then $Y = 0$. If $X > 0$, toss a fair coin repeatedly, all tosses are independent. Let $Y$ be the number of tails before the $X$th head.

1. (5 points) Find $E(2^X)$.
2. (5 points) Find $E(X!)$.
3. (5 points) Find $E(Y \mid X = 3)$.
4. (5 points) Find $E(Y)$.
Exam 4C

1. (5 points) If $X \sim \text{Exponential}(\lambda)$, find the moment generating function of $X$.

2. (5 points) Suppose that $W_k \sim \text{i.i.d Exponential}(5)$ for $k = 1, 2, \ldots, 100$. Let $S_{100} = \sum_{k=1}^{100} W_k$. Find a numerical approximation to $P(18 < S_{100} < 23)$.

3. (5 points) Suppose that $X_n \sim \text{Exponential}(n)$, for $n = 1, 2, 3, \ldots$. Show that $X_n \overset{P}{\to} 0$.

4. (5 points) Let $Y_n = (n + 1)X_n$ for $X_n$ defined as in the previous part, and let $Y \sim \text{Exponential}(1)$. Show that $Y_n \overset{D}{\to} Y$.

Exam 4D

1. (5 points) If $X \sim \text{Poisson}(\lambda)$, find the moment generating function of $X$.

2. (5 points) Suppose that $W_k \sim \text{i.i.d Poisson}(4)$ for $k = 1, 2, \ldots, 100$. Let $S_{100} = \sum_{k=1}^{100} W_k$. Find and evaluate an exact expression for $P(399 \leq S_{100} \leq 401)$. (Recall that sums of independent Poisson random variables are Poisson.)

3. (5 points) Let $Y \sim N(\mu, \sigma^2)$ where $\mu = E(S_{100})$ and $\sigma^2 = \text{Var}(E_{100})$. Use the central limit theorem to calculate $P(398.5 < Y < 401.5)$ as an approximation to $P(399 \leq S_{100} \leq 401)$.

4. (5 points) Suppose that $X_n \sim \text{Poisson}(1/n)$, for $n = 1, 2, 3, \ldots$. Show that $X_n \overset{P}{\to} 0$.

Exam 11B

In a game, a player begins with 2 chips and has the goal of obtaining 12 chips through a series of bets. In each bet, the player wagers an amount $s$ and either wins or loses, with the current fortune changing increasing or decreasing by $s$ accordingly. It is not allowed to bet more than the current fortune or to bet so that the fortune would exceed 12 if the bet is won. For some betting strategies, the amount bet can vary depending on the current fortune. Each individual bet is won with probability $\theta = 2/5$. The player is said to win if he achieves a fortune of 12 chips.

1. (4 points) If the player uses the bet-one strategy, the player bets one chip for each bet until the game ends. What is the probability of winning using this strategy?

2. (4 points) If the player uses the bet-two strategy, the player bets two chips for each bet until the game ends. What is the probability of winning using this strategy?

3. (4 points) If the player uses the bold strategy, the player bets everything if the current fortune is 6 or less, and bets just what is needed to get to 12 in a single bet if the current fortune exceeds 6. Find the probability of winning using this strategy.

4. (4 points) If the player uses the 3-6-12 strategy, the player bets just enough chips to reach the next possible target number from 3, 6, and 12, when possible. Otherwise (only if the current fortune is 1), the player bets everything. Find the probability of winning using this strategy.

5. (4 points) Suppose the player were offered an alternative single bet with different payoff odds: Bet two chips and win ten with probability 1/11, but lose two otherwise. Should the player accept this bet or take one of the previous four strategies (if he must bet until he wins 12 chips or goes broke)?