Discussion Problems, September 29, 2008

A coin with head probability $\theta = 0.2$ is tossed repeatedly. Identify the distribution of the random variable and the values of any parameters in addition to finding each probability.

1. Find the probability that exactly four of the first seven coin tosses are heads.
2. Find the probability that exactly six tails appear before the second head.
3. Find the probability that exactly six heads appear before the second tail.
4. Find the probability that ten or more tails appear before the first head.
5. Find the probability that the number of tails between the first and third head is exactly six.
6. Find the probability that the number of heads before the first tail is between one and five, inclusive.

Additional problems:

7. Two friends play a game taking turns tossing a fair coin. The first person to toss a head wins. What is the probability that the person who tosses first wins?
8. Find the value of $\theta$ that maximizes $P(X = 5)$ when $X \sim \text{Geometric}(\theta)$.
9. If $X \sim \text{Poisson}(4)$, find $P(X \geq 2)$.
10. Suppose that $X \sim \text{Geometric}(0.2)$ and $Y \sim \text{Geometric}(0.2)$. Suppose in addition that $X$ and $Y$ are independent, meaning that $P(X = x \cap Y = y) = P(X = x)P(Y = y)$ for all $x$ and $y$. Let $Z = X + Y$. Find $P(Z = 5)$.
11. Suppose that $X \sim \text{Negative-Binomial}(2, 0.2)$ and $Y \sim \text{Negative-Binomial}(3, 0.2)$. Suppose in addition that $X$ and $Y$ are independent, meaning that $P(X = x \cap Y = y) = P(X = x)P(Y = y)$ for all $x$ and $y$. Let $Z = X + Y$. Find $P(Z = 5)$. 