3. INTERPRETING A CONFIDENCE INTERVAL

Going back to the example on pp. 342–343, a simple random sample was taken to estimate the percentage of students registered at a certain university in fall, 1977, whose parents were both college graduates. An approximate 95%-confidence interval for this percentage ran from 75% to 83%, because

\[ \text{sample percentage } \pm 2 \text{ SE} = 75\% \text{ to } 83\%. \]

It seems more natural to say "There is a 95% chance that the population percentage is between 75% and 83%." But there is a problem here. In the frequency theory of chance, a chance represents the percentage of the time that something will happen. No matter how many times you take stock of all the students registered at that university in the fall of 1977, the percentage with parents who were both college graduates won't change. Either this percentage is between 75% and 83%, or it isn't. So there really isn't any way to define the chance that the parameter will be in this interval—or any other. That is why statisticians have to turn the problem around slightly. They realize that the chances are in the sampling procedure, not in the parameter, and they use the new word "confidence" to remind you of this.

So the confidence level of 95% has to say something about the sampling procedure, and we are going to see what this is. The first point to notice is
that the confidence interval depends on the sample. If the sample had come out differently, the confidence interval would have been different, because the percentages in the sample would have been different. With some samples, the interval "sample percentage ± 2 SE" does trap the population percentage. (The word statisticians use is cover.) But with other samples, the interval fails to cover. It's like buying a used car. Sometimes you get a lemon—a confidence interval which doesn't cover the parameter.

Three confidence intervals

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covers a lemon another lemon
|x|   |   |
x   x
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$x =$ the population percentage

The interval "sample percentage ± 2 SE" has a confidence level of 95%. This can now be interpreted: For about 95% of all samples, the interval "sample percentage ± 2 SE" covers the population percentage, and for the other 5% it fails.

Of course, an investigator usually can't tell whether or not his particular interval covers the population percentage: he doesn't know this parameter, it's what he is trying to estimate. But he followed a procedure which works 95% of the time—taking a simple random sample, and then going two SEs either way from the sample percentage. It is as if his interval was drawn at random from a box full of intervals, of which 95% cover the parameter, and only 5% are lemons. It beats buying a used car.

This interpretation of confidence levels is a bit difficult, because it involves thinking not only about the actual sample, but about other samples that could have been drawn. The interpretation is illustrated in Figure 1. A hundred survey organizations are hired to estimate the percentage of red marbles in a large box. Unknown to the organizations, this percentage is 80%. Each organization takes a simple random sample of 2,500 marbles, and computes a 95%-confidence interval for the percentage of reds in the box, using the formula "percentage of reds in sample ± 2 SE." The percentage of reds is different from sample to sample, and so is the estimated standard error. As a result, the confidence intervals have different centers and lengths. Some of the organizations get intervals covering the percentage of red marbles in the box, others fail. In the figure, these intervals are drawn at different heights, so you can tell them apart. About 95% of them should cover the percentage of red marbles in the box, marked by a vertical line. And in fact, 95 out of 100 do.
Figure 1. Interpreting confidence intervals. The 95%-confidence interval is shown for a hundred different samples. The interval changes from sample to sample. For about 95% of the samples, the interval covers the population percentage, marked by a vertical line.