In the notes from last lecture we finished up the discussion of **maximum error estimates, confidence intervals, sample size determination** and **maximum likelihood estimates** from sections 7.1 and 7.2 written up in last week's notes.

Hypothesis testing Sections 7.3 and 7.4 of text :
A type I error refers to rejecting the **null hypothesis** H_0 when H_0 is true. A type II error means we accept H_0 when H_0 is false (so that the alternative hypothesis H_a (also written H_1) is true
We have the table

		Accept H_0	Reject H_0
H_0	true	correct decision	type I error
	false	type II error	correct decision

Typically the **alternative hypothesis** represents the claim that we wish to establish.
The type I error probability is also called the **significance level** of the hypothesis test.

For a **two sided test** of the null hypothesis $H_0: \mu = \mu_0$ versus $H_a: \mu \neq \mu_0$ using a standard normal Z variable the **rejection region** (also called the **critical region**) is the extreme region

$$Z > z_{\alpha/2} \text{ or } Z < -z_{\alpha/2}$$

having small total probability (area) α under the standard Z curve. For a **one sided test** the rejection region, also of total area (probability) α for an alternative hypothesis of the form $\mu > \mu_0$ is

$$Z > z_\alpha \quad (\text{ or } Z < -z_\alpha \text{ when the alternative is } \mu < \mu_0)$$

EXAMPLE 1 Problem 7.29 Change the rejection region of the paint drying example in the text to $\bar{x} > 20.50$ (instead of 20.75) We still have sample size *n* = 36, $\sigma = 2.4$ minutes , $H_0: \mu = 20$

a)

$$\alpha = P(type\ I\ error) = P(Z = \frac{\bar{X} - 20}{2.4/\sqrt{36}} > \frac{20.50 - 20}{2.4/6} = \frac{3}{2.4} = 1.25) = P(Z > 1.25) = P(Z < -1.25) = .1056$$

b)

$$\beta = P(type\ II\ error\ when\ \mu = 21) = P(accept\ H_0\ when\ \mu = 21) = P(\bar{x} < 20.50)$$

$$= P(Z = \frac{\bar{X} - 21}{2.4/6} < \frac{20.50 - 21}{2.4/6} = -1.25) = \alpha = .1056$$

so in this example the type I error probability α equals the type II error probability β .

EXAMPLE 2 Problem 7.31 of the text (to which we've added parts c) and d) to this problem)
The sample size of the paint drying example of the text has been changed to *n* = 50 (instead of *n* = 36)
Reject $H_0: \mu = 20$ if $\bar{x} > 20.75$

a) $$\alpha = P(Z = \frac{\bar{X} - 20}{2.4/\sqrt{50}} > \frac{20.75 - 20}{2.4/\sqrt{50}} = \frac{.75\sqrt{50}}{2.4} = 2.2097) = P(Z < -2.2097) = .0136 \approx .014 \text{ or } 1.4\%$$

b)

$$\beta = P(\bar{x} < 20.75 \text{ when } \mu = 21) = P(Z = \frac{\bar{x} - 21}{2.4/\sqrt{50}} < \frac{-.25}{2.4/\sqrt{50}}) = P(Z < -.73656) = .231$$

c) (like 7.33b) What can we say about the type I error probability α if we change the null hypothesis from

$$H_0\!: \mu = 20 \quad \text{to} \quad H_0\!: \mu \le 20 \ .$$

In this case we can not evaluate the probability α exactly unless we specify a particular value μ_0 of the population mean $\mu = \mu_0 \le 20$. However what we can say is that this probability will be less than or equal to its value at the borderline case $\mu_0 = 20$ already found in part a) to be 1.4%. This is because shifting the mean further to the left of 20 makes it more difficult to land in the rejection region $\bar{x} > 20.75$ (more extreme). Thus if $\mu = \mu_0 \le 20$ we can say

$$\alpha = P\left(Z = \frac{\bar{X} - \mu}{2.4/\sqrt{50}} = \frac{\bar{X} - 20}{2.4/\sqrt{50}} + \frac{20 - \mu}{2.4/\sqrt{50}} > \frac{.75\sqrt{50}}{2.4} + \frac{20 - \mu}{2.4/\sqrt{50}}\right) \ \le \ P\left(Z > \frac{.75\sqrt{50}}{2.4}\right) = .014$$

as claimed.

d) (like 7.32) For what values of $\bar{x}$ must we reject the null hypothesis $H_0\!: \mu = 20$ if the significance level (type I error probability) is $\alpha = .005$ (half of one percent) instead of 1.4% as found in part a) ?

$$.005 = P\left(Z = \frac{\bar{X} - 20}{2.4/\sqrt{50}} > z_{.005} = 2.575\right) = P\left(\bar{X} > 20 + 2.575(2.4/\sqrt{50}) = 20.874\right)$$

Thus the rejection region is now $\bar{X} > 20.87$ rather than $\bar{X} > 20.75$.

EXAMPLE 3 (like 7.30) For a 2-sided test example : A process for making ball bearings is under control if the diameter of a bearing has a mean of μ = .30 cm and a standard deviation of σ = .003 cm. A sample of size n = 36 is taken each day and the null hypothesis $H_0\!: \mu = .30$ is rejected if the sample mean is much less than or is much more than the population mean .30 in the sense that either extreme

$$\bar{x} < .299 \ \text{ or } \ \bar{x} > .301$$

is realized. To see that this really is extreme behavior for a sample of size 36 when the standard deviation is .003, note that the above rejection region is equivalent to

$$Z = \frac{\bar{x} - .30}{.003/\sqrt{36}} < \frac{-.001}{.003/6} = -2, \ \text{ that is } Z < -2 \text{ or } Z > 2 \text{) } \ .$$

a)

$$\alpha = P(Z < -2 \ \text{ or } \ Z > 2) = 2P(Z < -2) = 2(.0228) = .0456 \ \text{ or } \ 4.56\% \ .$$

b)

$$\beta = P(\text{type II error when } \mu = .302) = P(.299 < \bar{X} < .301) = P\left(-6 < Z = \frac{\bar{X} - .302}{.003/\sqrt{36}} < -2\right)$$

$$\approx \ P(Z < -2) = .0228 \ .$$

EXAMPLE 4 Greener Pastures Dairy claims its low fat swiss cheese has a mean fat content of under 20 grams per 8 oz serving. You have been hired by the Dairy board to test this claim. What null hypothesis and what alternative hypothesis do you use ?

The alternative hypothesis should correspond to the claim you want to establish. In this case we test the null hypothesis which is the borderline case that the mean equals 20 (null hypothesis) against the alternative that it is less than 20 :

$$H_0\!: \mu = 20 \ \text{ against } \ H_a\!: \mu < 20 \ .$$