Stat 992: Lecture 38
Image Registration I.

Moo K. Chung mchung@stat.wisc.edu

April 30, 2004

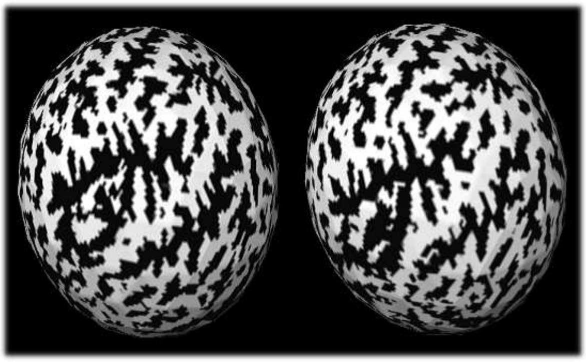

Figure 1: Sulcal patterns of brain at age 14 (left) and at age 19 (right). The global patterns are similar but there is a pattern variation present. The question is if these pattern difference is statistically significant.

1. *Similarity measure.* Consider two geometric objects Ω_1 and Ω_2 in images. We are interested in testing if the shape of two objects Ω_1 and Ω_2 are statistically different. This requires quantifying the shape variations via the concept of curve and image registration and template. Let I_i be the image intensity that defines the geometric object Ω_i. In order to compare image intensities across images, we need similarity metric that measures the closeness of two images. One very simple similarity metric would be the local sample intensity correlation ρ defined as

$$\rho(I_1(x), I_2(x)) = \frac{\mathbf{Cov}(I_1(B_x), I_2(B_x))}{\mathbf{Var}^{1/2} I_1(B_x) \mathbf{Var}^{1/2} I_2(B_x)},$$

where B_x is a small ball centered around x. Other similarity measure such as the *mutual information* have been used. We have computed sample intensity correlation for both 5×5 and 10×10 windows (Figure 3). Some regions show extremely lower correlation indicating those regions are not matching anatomically. Then the question is if we can increase the anatomical matching by deforming image. This is the basic idea of intensity-based image registration

2. *Image deformation.* Given image intensity $I_1(x)$, $x \in \Omega_1$, the deformation of image via displacement field $u = (u_1, \cdots, u_n)$ to be $I_(x + u(x)), x \in \Omega_1$. Then we will increase the matching of two images by increasing the total similarity measure,

$$\int \rho^2(I_1(x + u(x)), I_2(x)) \; dx.$$

One may use L_2 norm such that the total similarity measure would be

$$\int [I_1(x + u(x)) - I_2(x)]^2 \; dx.$$

Since planer curves can be characterized by the curvature function, Thomas Hoffmann use the curvature difference as the similarity measure in the curve registration problem, i.e.

$$\int [\kappa_1(x + u(x)) - \kappa_2(x)]^2 \; dx.$$

However the minimization of the above similarity will result in not very smooth displacement field. So we need to introduce a measure of roughness. The usual measure of roughness is to use the first derivative of the displacement

$$\|\partial u\| = \sum_{i,k} \Big| \frac{\partial u_k}{\partial x_i} \Big|^2$$

or the second derivatives

$$\|\partial^2 u\| = \sum_{i,j,k} \Big| \frac{\partial^2 u_k}{\partial x_i \partial x_j} \Big|^2.$$

Then we maximize the similarity while minimizing the roughness, i.e.

$$\hat{u} = \arg\max_u \int \rho(I_1(x+u), I_2) \; dx - \lambda \int \|\partial^2 u\|^2 \; dx$$

where $\lambda > 0$. Most of research in image registration have been concentrated on choosing proper similarity and roughness measures. See Brain Warping by Toga and Functional Data Analysis by Ramsay and Silverman for an overview of image and curve registration problem.

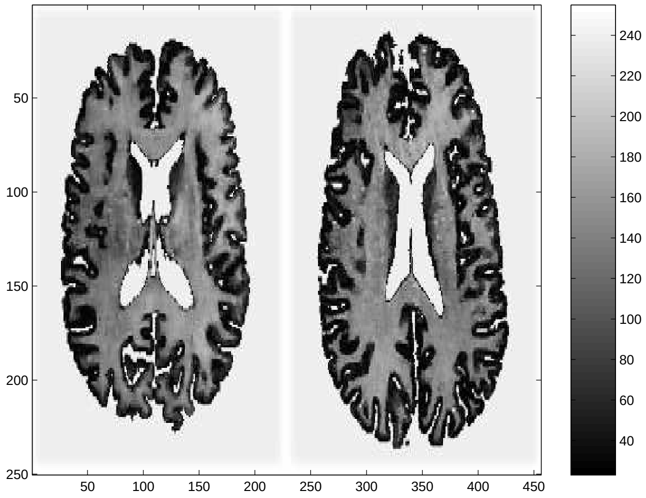

Figure 2: Brain cross-sections of two different subjects

3. *Shape inference.* Under the null assumption of no shape difference between two geometric objects, the displacement fields would be small. So it would be reasonable to model the displacement field stochastically:

$$u(x) = \mu(x) + \Sigma^{1/2}(x)\epsilon(x).$$

where ϵ is the zero mean random vector field whose components are i.i.d. and Σ is the covariance matrix accounting for the spatially varying correlated noise. See

`http://www.math.mcgill.ca/chung/deformation/ni_deformation.pdf`

for detail. This model has been applied to the detection of brain tissue growth in a group of 28 children.

The hypothesis of interest is

$$H_0 : \mu(x) = 0 \text{ for all } x$$

vs.

$$H_1 : \mu(x) \neq 0 \text{ for some } x.$$

The reasonable test statistic would be

$$\int \epsilon'(x)\Sigma(x)\epsilon(x)\,dx.$$

It can be shown that the test statistic is approximately a $c\chi^2$-distribution (Chung, 2001) where the degrees of freedom and proportionally d is estimated by the method of moment matching. This is a global inference. For a local inference, we take $\sup_{x\in\Omega_1} \epsilon'(x)\Sigma(x)\epsilon(x)$ to be the test statistic.

If there are two groups of geometric objects: n objects Ω_i embedded in image I_i and m objects Ω'_i embedded in image I'_i. We can model the displacement of the first group

$$u_i(x) = \mu_1(x) + \Sigma_1^{1/2}(x)\epsilon_i(x)$$

and those of the second group

$$u'_j(x) = \mu_2(x) + \Sigma_1^{1/2}(x)\epsilon_j(x).$$

where ϵ_i would be zero mean i.i.d. Gaussian random vector fields and u_i and u'_j are the displacement to a template. If we use one subject as a template, effectively we are losing one degree of freedom. So for small sample, it would be better to register to the independent template. Then we would be interested in testing if the group means are identical:

$$H_0 : \mu_1(x) = \mu_2(x) \text{ for all } x.$$

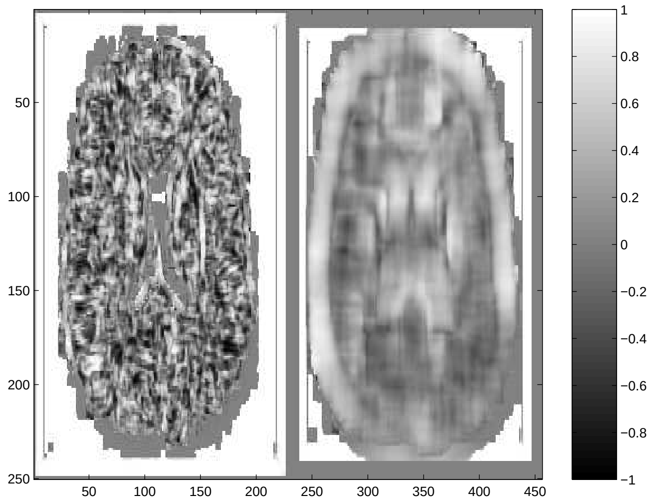

Figure 3: local intensity correlation in 5×5 window (left) and 10×10 window (right). By image registration, it is needed to increase the correlation uniformly in the image.