

Stat 992: Lecture 19

Multiple comparisons II.

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1. *Bonferroni correction.* One standard method for dealing with multiple comparison is to use the Bonferroni correction. Note that the probability measure is additive, i.e.

$$P\left(\bigcup_{j=1}^m E_j\right) \leq \sum_{j=1}^m P(E_j). \quad (1)$$

This inequality and its variations are called Bonferroni inequalities and it has been used in the construction of simultaneous confidence intervals and multiple comparisons when the number of hypotheses are small. In the previous lecture, the type I error was given by

$$\alpha = P\left(\bigcup_{j=1}^m \{Y(x_j) > h\} \mid \mathbb{E}Y = 0\right) \quad (2)$$

$$\leq \sum_{j=1}^m P(Y(x_j) > h \mid \mathbb{E}Y = 0) \quad (3)$$

So by controlling each type I error separately so that

$$P(Y(x_j) > h \mid \mathbb{E}Y = 0) < \alpha/m,$$

we can construct level α test. For image of size 95×68 , there are $m = 95 \times 68 = 6460$ hypotheses to test. So $\alpha/m = 0.05/6460 = 7.7 \times 10^{-6}$. For $Y(x_j) \sim N(0, 1)$, the corresponding threshold is computed by

```
>> norminv(1-7.7*10^(-6), 0, 1)
ans =
    4.3229
```

By setting $h = 4.33$, any image intensity larger than the threshold is taken as statistically significant signal. The problem with the Bonferroni correction is that in most cases it is considerably too conservative since each pixel or voxel intensities are strongly correlated and in turn it gives highly correlated statistics across pixels. In some cases, the size of image tend to be $200 \times 200 \times 100$, which gives extremely high threshold of 5.57.

```
>> norminv(1-0.05/4000000, 0, 1)
ans =
    5.5733
```

See Miller, R. G. Jr. *Simultaneous Statistical Inference*. New York: Springer-Verlag, 1991.

Problem 29. Based on slightly different version of Bonferroni inequalities, it is possible to have a slightly better inequality than (3) so that the threshold h is lower than the above Bonferroni correction. Come up with a better less conservative method than the one presented in this lecture.

2. *Random fields approach.* Assume $\mathbb{E}Y = 0$.

$$\begin{aligned} \alpha(h) &= P\left(\bigcup_{j=1}^m \{Y(x_j) > h\}\right) \\ &= 1 - P\left(\bigcap_{j=1}^m \{Y(x_j) \leq h\}\right) \\ &= 1 - P(\max_{x_j \in \Omega} Y(x_j) \leq h) \\ &= P(\max_{x_j \in \Omega} Y(x_j) > h) \end{aligned}$$

So in order to construct α -level test, we need to know the distribution of the maximum of m correlated Gaussian random variables (see HW problem 28). For continuous mean zero random fields Y , following the similar argument, we get

$$\begin{aligned} \alpha(h) &= P\left(\bigcup_{x \in \Omega} \{Y(x) > h\}\right) \\ &= P(\sup_{x \in \Omega} Y(x) > h). \end{aligned}$$

Analytically computing the exact distribution of the maxima of random fields is hard. Multiple testing procedures based on the distribution of the maxima of fields is usually called random field approach and the P -value based on the maximum field is called *corrected P-value* to distinguish it from uncorrected P -value.

Let $Z = \sup_{x \in \Omega} Y(x)$ and F_Z to be the cumulative distribution of Z . Then given $\alpha = 0.05$, we compute $h = 1 - F_Z^{-1}(\alpha)$. Then the region of statistically significant signal is given by $\{x \in \Omega : Y(x) > h\}$.

Homework 4. Solve Problems 24-32 and any other previous problems where complete solutions are not given. Due March 22 Monday 11:00am.