Stat 312: Lecture 9
Chi-squared distributions

Moo K. Chung
mchung@stat.wisc.edu

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Concepts

1. Let $X_1, \cdots, X_n$ be a random sample from $N(\mu, \sigma^2)$.

$$\frac{(n-1)S^2}{\sigma^2} = \frac{1}{\sigma^2}\sum_{j=1}^{n}(X_j - \bar{X})^2 \sim \chi^2_{n-1}$$

2. Critical values for χ^2_n:

$$P(\chi^2_{1-\alpha/2,n} < \chi^2_n < \chi^2_{\alpha/2,n}) = 1 - \alpha$$

3. $100(1-\alpha)\%$ CI for σ^2:

$$\frac{(n-1)}{\chi_{\alpha/2,n-1}}s^2 < \sigma^2 < \frac{(n-1)}{\chi_{1-\alpha/2,n-1}}s^2$$

4. $X \sim \exp(\lambda)$: exponential distribution with parameter λ. The density function is defined for $x \geq 0$.

$$f(x) = \frac{1}{\lambda}e^{-x/\lambda}$$

In-class problems

Continuing Example 7.12. Fat content of 10 randomly selected hot dogs. Assuming that observations come from normal distribution, find a 95% CI for the population variance of fat content.

```
> x<-c(25.2,21.3,22.8,17.0,29.8,21.0,
       25.5,16.0,20.9,19.5)
> sd(x)
[1] 4.13414
> qchisq(0.975,9)
[1] 19.02277
> qchisq(0.025,9)
[1] 2.700389
> 9/qchisq(0.025,9)*sd(x)
[1] 13.77848
> 9/qchisq(0.975,9)*4.13414
[1] 1.955933
```

Review problem

Lifetime of an electrical component has an exponential distribution with parameter λ. Estimate λ by matching moments.

```
> x<-c(41.53,18.73,2.99,30.34,12.33,117.52,
73.02,223.63,4.00,26.78)
> mean(x)
[1] 55.087
```

Obtain the maximum likelihood estimate of the probability that lifetime exceeds 10.

```
> exp(-10/55.09)
[1] 0.8340006
```

Suppose that there are 100 electrical components with $\bar{x} = 55$, $s = 40$. Find a 95% confidence interval for λ.

```
> 55+qnorm(0.975)*40/10
[1] 62.83986
> 55-qnorm(0.975)*40/10
[1] 47.16014
```

Self-study problems

Example 7.15., Exercise 7.43.