Stat 312: Lecture 8
Student's t-distribution

Instructor's valentine gift to students :)
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Concepts

1. When n large,
$$Z \sim \frac{\bar{X}-\mu}{S/\sqrt{n}} \sim N(0,1).$$
2. What if n is small?
$$T \sim \frac{\bar{X}-\mu}{S/\sqrt{n}} \sim t_{n-1},$$
a t distribution with $n-1$ degrees of freedom (Review STAT311).
3. As $n \to \infty$, $t_n \to N(0,1)$.
4. Critical values:
$$P(-t_{\alpha/2,n-1} < T < t_{\alpha/2,n-1}) = 1-\alpha.$$
5. Suppose $X_1, \cdots, X_n \sim N(\mu, \sigma^2)$, where μ and σ are unknown. $100(1-\alpha)$ CI for μ is
$$\bar{x} \pm t_{\alpha/2} s/\sqrt{n}.$$

In-class problems

```
> x<- -50:50/10
> plot(x,dt(x,1000),type='l')

> qt(.975, df = c(1:10,20,100,1000,10000))
 [1] 12.706205  4.302653  3.182449
 [4]  2.776445  2.570582  2.446912
 [7]  2.364624  2.306004  2.262157
[10]  2.228139  2.085963  1.983972
[13]  1.962339  1.960201
> qnorm(0.975)
[1] 1.959964
> pt(2.228,10)
[1] 0.9749941
```

Example 7.12. Fat content of 10 randomly selected hot dogs. Assuming that observations come from normal distribution, find a 95% CI for the population mean fat content.

```
> x<-c(25.2,21.3,22.8,17.0,29.8,21.0,
       25.5,16.0,20.9,19.5)
> sd(x)
[1] 4.13414
> mean(x)+qt(0.975,9)*sd(x)/sqrt(10)
[1] 24.85739
> mean(x)-qt(0.975,9)*sd(x)/sqrt(10)
[1] 18.94261
```

Assuming $X_i \sim N(\mu, \sigma^2 = 4.13^2)$, find a 95% CI for μ.

```
> mean(x)+qnorm(0.975)*4.13/sqrt(10)
[1] 24.45975
> mean(x)-qnorm(0.975)*4.13/sqrt(10)
[1] 19.34025
```

Self-study problems

Exercise 7.29., 7.33.

Assignment III.

Due Feb 27, 11:00am.
Exercise 7.32, 7.34., 7.42., 7.44., 8.10.
The first midterm will cover chapters 6,7 and 8.1. Sample midterm problems will be posted on Feb. 25.