Stat 312: Lecture 5
Confidence Intervals

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February 3, 2003

Concepts

1. For population parameter θ, 95% confidence interval $(\hat{\theta}_L, \hat{\theta}_U)$ of θ is computed by solving

$$P(\hat{\theta}_L \leq \theta \leq \hat{\theta}_U) = 0.95$$

2. Let $X_i \sim N(\mu, \sigma^2)$ with known σ^2 and unknown μ. 95% *confidence interval* for μ is

$$\hat{\mu}_L = \bar{x} - 1.96 \cdot \sigma/\sqrt{n},\ \hat{\mu}_U = \bar{x} + 1.96 \cdot \sigma/\sqrt{n}.$$

3. Let $X_i \sim N(\mu, \sigma^2)$ with known σ^2 and unknown μ. $100(1-\alpha)\%$ confidence interval for μ is.

$$\hat{\mu}_L = \bar{x} - z_{\alpha/2} \cdot \sigma/\sqrt{n},\ \hat{\mu}_U = \bar{x} + z_{\alpha/2} \cdot \sigma/\sqrt{n}.$$

In-class problems

Continuing Exercise 6.25. Assuming $\sigma = 18.86$, find 95% confidence interval of μ.

```
> b<-qnorm(0.975,0,sigma/sqrt(10))
> a<-qnorm(0.025,0,sigma/sqrt(10))
> b
[1] 11.68933
> a
[1] -11.68933
> mean(X)-b
[1] 372.7107
> mean(X)-a
[1] 396.0893

> qnorm(0.995)
[1] 2.575829
> qnorm(0.99)
[1] 2.326348
> qnorm(0.975)
[1] 1.959964
> qnorm(0.95)
[1] 1.644854
```

Self-study problems

Example 7.2.,7.3.,7.4.,7.5.

Assignment II.

Due Feb 13. 11:00am.
Exercise 6.30., 7.4., 7.10., 7.14., 7.22., 7.26.

Bonus Problem (Hard). Let $X_1, \cdots, X_n$ be a random sample with mean μ and variance σ^2. Among all estimators of the form $\hat{\mu} = \sum_{j=1}^{n} c_j X_J$, find MVUE and prove that it is in fact MVUE. Hand in this problem directly to the instructor *during the office hour*. No partial credit for this problem.

Midterms

One page note and a calculator are allowed.
Midterm I. March 4, 11:00am-12:15pm (Tuesday).
Midterm II. April 15, 11:00am-12:15pm (Tuesday).
4 problems will be given: 1 easy, 2 medium difficulty and 1 challenging problems. Mock midterm problems will be posted on the WEB one week before the midterms.