Stat 312: Lecture 4
Maximum Likelihood Estimation

Moo K. Chung
mchung@stat.wisc.edu

January 30, 2003

Concepts

1. For a random sample $X_1, \cdots, X_n$ the *likelihood function* is given as the product of probability or density functions, i.e. $L(\theta) = f(x_1;\theta)f(x_2;\theta)\cdots f(x_n;\theta)$.

2. The *maximum likelihood estimatate* of θ maximizes $L(\theta)$. If we denote $\hat{\theta} = \theta(x_1, \cdots, x_n)$ to be the maximum likelihood estimate, The *maximum likelihood estimator* (MLE) of θ is $\hat{\theta} = \hat{\theta}(X_1, \cdots, X_n)$. **Note:** x_i are numbers while X_i are random variables.

3. When the sample size is large, the maximum likelihood estimator of θ is approximately unbiased. The MLE of θ is approximately the MVUE of θ. This is why it is the most widely used parameter estimation technique.

4. If explicit density function is not available, you can not apply MLE. In this case apply the method of moment matching.

5. (Invariance Principle) If $\hat{\theta}_1, \hat{\theta}_2$ are the MLE's of θ_1, θ_2, the MLE of $h(\theta_1, \theta_2)$ is $h(\hat{\theta}_1, \hat{\theta}_2)$.

In-class problems

Example 6.15. Example 6.16. Example 6.21.
Exercise 6.25. Assuming normal distribution, find c such that $P(\text{strength} \leq c) = 0.95$.

```
> x<-strength
> length(x)
[1] 10
> sd(x)
[1] 19.87852
```

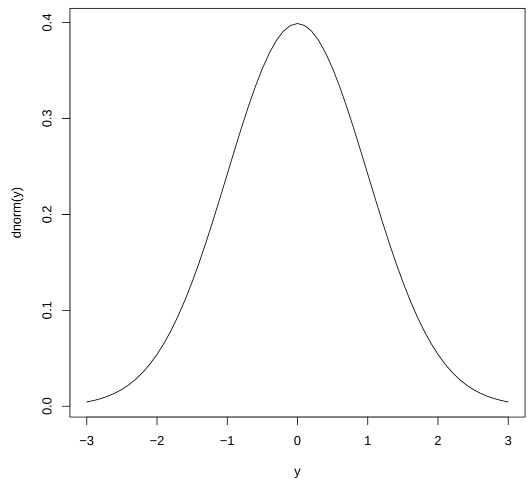

Figure 1: Density of $N(0, 1)$

```
> sigma<-sqrt((length(x)-
1)/length(x))*sd(x)
> sigma
[1] 18.85842
> qnorm(0.95,mu,sigma)
[1] 415.4193
> qnorm(0.95)
[1] 1.644854
> pnorm(415,mu,sigma)
[1] 0.9476644
> y<- -30:30/10
> plot(y,dnorm(y),'l')
```

Self-study problems

Example 6.17., Example 6.18., Exercise 6.23., Exercise 6.29.