

Stat 312: Lecture 13

Two-sample tests

Moo K. Chung
mchung@stat.wisc.edu

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Concepts

1. The P -value is the smallest level of significance at which H_0 would be rejected.
 $P\text{-value} \leq \alpha \rightarrow$ reject H_0 at level α .
 $P\text{-value} > \alpha \rightarrow$ do not reject H_0 at level α .
The smaller the P -value, the easier to reject H_0 .
2. Let X_1, \dots, X_n and Y_1, \dots, Y_m be two independent samples with mean μ_X, μ_Y and variance σ_X^2, σ_Y^2 respectively. Assume n and m to be large. The test statistic for testing

$$H_0 : \mu_X = \mu_Y \text{ vs. } H_1 : \mu_X \neq \mu_Y$$

$$Z = \frac{\bar{Y} - \bar{X}}{\sqrt{\sigma_X^2/n + \sigma_Y^2/m}} \sim N(0, 1).$$

3. Let X_1, \dots, X_n and Y_1, \dots, Y_m be two independent samples with $X_i \sim N(\mu_X, \sigma_X^2)$ and $Y_j \sim N(\mu_Y, \sigma_Y^2)$. Then we can show that

$$Z = \frac{\bar{Y} - \bar{X}}{\sqrt{\sigma_X^2/n + \sigma_Y^2/m}} \sim N(0, 1).$$

In-class problems

Example 8.11. 47 out of 102 doctors did not know the generic name for the drug methadone. Let p be the proportion of doctors who knew the generic name. Determine the P -values for testing

$$H_0 : p = 0.5, \text{ vs } H_1 : p < 0.5$$

```
> p<-47/102
> (p-0.5)/sqrt(0.5*0.5/102)
[1] -0.792118
> pnorm(-0.79)
[1] 0.2147639
```

So we do not reject H_0 at any level less than 0.2.

Example 1. A study was conducted to compare the reaction times of men and women to a stimulus. 50 men and 65 women were employed in the experiment. Let X_i be the reaction time of the i -th man and Y_j be the reaction time of the j -th woman. The following measurements were taken: $\bar{x} = 3.6$ seconds, $\bar{y} = 3.8$ seconds, $s_X^2 = 0.18$, $s_Y^2 = 0.14$. Is there any difference between the reaction times of men and women?

```
> z=(3.6-3.8)/sqrt(0.18/50+0.14/65)
> z
[1] -2.63664
> qnorm(0.025)
[1] -1.959964
```

Example 2. 12 men and 9 women were employed in the experiment. Let X_i be the reaction time of the i -th man and Y_j be the reaction time of the j -th woman. There is a reason to believe that $X_i \sim N(\mu_X, 0.18)$ and $Y_j \sim N(\mu_Y, 0.14)$. The following measurements were taken: $\bar{x} = 3.6$ seconds, $\bar{y} = 3.8$ seconds. Is there any difference between the reaction times of men and women?

```
> z=(3.6-3.8)/sqrt(0.18/12+0.14/9)
> z
[1] -1.144155
```

What is the main reason you are getting the different results in Example 1 and 2?

Self-study problems

Exercise 8.53. Determine the P -value in Example 1 and 2. Example 9.1., 9.4., 9.5.