Stat 312: Lecture 11
Testing on Population Means

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Concepts

1. $P(\text{Type I error}) = P(\text{reject } H_0|H_0 \text{ true}) = \alpha$. The resulting α is called the *significance level* of the test and the corresponding test is called a *level α test*. We will use test procedures that give α less than a specified level (0.05 or 0.01 usually).

2. Testing mean μ with known variance σ^2.

$$H_0 : \mu = \mu_0 \text{ vs. } H_1 : \mu < \mu_0$$

   Test statistic: $z = \frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}}$. Rejection region for level α test: $z \leq -z_\alpha$.

3. Testing mean μ with unknown variance σ^2.

$$H_0 : \mu = \mu_0 \text{ vs. } H_1 : \mu < \mu_0$$

   Test statistic: $t = \frac{\bar{x}-\mu_0}{s/\sqrt{n}}$. Rejection region for level α test: $z \leq -t_{\alpha,n-1}$.

4. Testing mean μ when the sample size is large.

$$H_0 : \mu = \mu_0 \text{ vs. } H_1 : \mu < \mu_0$$

   Test statistic: $z = \frac{\bar{x}-\mu_0}{s/\sqrt{n}}$. Rejection region for level α test: $z \leq -z_\alpha$.

In-class problems

Assume the IQ of a dog follows $X_i \sim N(\mu, 10^2)$. The IQ of 10 dogs are measured: 30, 25, 70, 110, 40, 80, 50, 60, 100, 60. We want to test if dogs are as smart as people by testing

$$H_0 : \mu = 100 \text{ vs. } H_1 : \mu < 100$$

at level $\alpha = 0.05$.

```
> x<-c(30, 25, 70, 110, 40, 80, 50, 60,
100, 60)
> mean(x)
[1] 62.5
> z<-(mean(x)-100)/(10/sqrt(10))
> z
[1] -11.85854
> qnorm(0.05)
[1] -1.644854
```

Assume the IQ of a dog follows $X_i \sim N(\mu, \sigma^2)$, where σ is unknown. Test

$$H_0 : \mu = 100 \text{ vs. } H_1 : \mu < 100$$

at level $\alpha = 0.05$.

```
> t=(mean(x)-100)/(sd(x)/sqrt(10))
> t
[1] -4.205156
> qt(0.05,9)
[1] -1.833113
> t.test(x,mu=100,alternative="less",
 conf.level=0.95)
        One Sample t-test
 data:  x t = -4.2036, df = 9,
 p-value = 0.001147
 alternative hypothesis: true mean is
 less than 100
 95 percent confidence interval:
    -Inf 78.8531
 sample estimates: mean of x 62.5
```

Self-study problems

Example 8.6., 8.8., 8.9.

Assignment III.

Due March 13. 11:00am. 8.24., 8.26, 8.28., 8.36. (a) and (b) only, 8.38.