Stat 312: Lecture 10
Hypothesis testing

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Concepts

1. The *null hypothesis* H_0 is a claim about the value of a population parameter. The *alternate hypothesis* H_1 is a claim opposite to H_0.

2. A *test of hypothesis* is a method for using sample data to decide whether to reject H_0. H_0 will be assumed to be true until the sample evidence suggest otherwise.

3. A *test statistic* is a function of the sample data on which the decision is to be based.

4. A *rejection region* is the set of all values of a test statistic for which H_0 is rejected.

5. *Type I error*: you reject H_0 when H_0 is true. $P(\text{Type I error}) = P(\text{reject } H_0 | H_0 \text{ true}) = \alpha$. The resulting α is called the *significance level* of the test and the corresponding test is called a *level α test*. We will use test procedures that give α less than a specified level (0.05 or 0.01).

In-class problem

I believe that dogs are as smart as people. Assume IQ of a dog follows $X_i \sim N(\mu, 10^2)$. IQ of 10 dogs are measured: 30, 25, 70, 110, 40, 80, 50, 60, 100, 60. We want to test if dogs are as smart as people by testing

$$H_0 : \mu = 100 \text{ vs. } H_1 : \mu < 100.$$

One reasonable thing one may try is to see how high the sample mean is.

```
> x<-c(30, 25, 70, 110, 40, 80, 50, 60,
100, 60)
> mean(x)
[1] 62.5
```

Since the average IQ of 10 dogs are lower than 100, one would be inclined to reject H_0.

Let $\bar{X}$ be a test statistic and $R = (-\infty, 90]$ to be a rejection region. Let's compute the probability of making Type I error based on this testing procedure. Under the assumption H_0 is true,

$$X_i \sim N(100, 10^2).$$

Under this condition, $\bar{X} \sim N(100, 10)$ and

$$\alpha = P(\bar{X} \leq 90).$$

```
> pnorm(90,100,sqrt(10))
[1] 0.0007827011
```

By using this test procedure, it is highly unlikely to make Type I error. Let's see what happens when we change the rejection region.
When $R = (-\infty, 95]$, $\alpha = P(\bar{X} \leq 95)$.

```
> pnorm(95,100,sqrt(10))
[1] 0.05692315
```

When $R = (-\infty, 99]$, $\alpha = P(\bar{X} \leq 99)$.

```
> pnorm(99,100,sqrt(10))
[1] 0.3759148
```

The test procedure based on rejecting H_0 if $\bar{X} \leq 99$ will produce huge Type I error. Why?

Self-study problems

Example 8.1., 8.2., 8.3., 8.4., 8.5. Do not compute β.