# Stat312: Sample Midterm I.

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Answer all questions clearly and circle your final answer. If you submit two contradicting computations and answers, the grade will be the minimum of two. Correct answers without valid computation or explanation will not get full credit. If you used a wrong formula without much explanation of what you are doing and ended up submitting a wrong solution, you will get almost nothing.

1. Let $X_1, \cdots, X_n$ be a random sample from a normal distribution with mean $\mu$ and variance $\sigma^2$.
   (a) What is $\mathbb{E}(S^2/\sigma^2)$? $S^2$ is the sample variance. Explain your result (5 points).
   (b) Among all estimators of the form $aX_1 + bX_2$, find the minimum variance unbiased estimator of $2\mu$ (10 points).

2. Consider the sample of fat content of 10 randomly selected dogs: 25, 21, 22, 17, 29, 25, 16, 20, 19, 22. Suppose that these are from a normal population.
   (a) Find the sample standard deviation using two numbers 216 and 4806 in R output. No point will be given if you do not use these two numbers (5 points).
   (b) Find a $95\%$ confidence interval of the mean fat content (5 points).
   (c) Find a $95\%$ confidence interval of the standard deviation of fat content (5 points).

```
>X<-c(25,21,22,17,29,25,16,20,19,22)
>sum(X)
[1] 216
>sum(X^2)
[1] 4806
> qnorm(c(0.025,0.05))
[1] -1.96 -1.64
> qt(0.025,c(9,10))
[1] -2.26 -2.23
> qchisq(0.025,c(9,10))
[1] 2.70 3.25
> qchisq(0.975,c(9,10))
[1] 19.02277 20.48318
```

3. Suppose you tossed $n = 200$ biased coins and observed 140 heads.
   (a) Compute a $98\%$ confidence interval for the probability of getting head (10 points).
   (b) Compute a $98\%$ confidence interval for the following ratio: probability of getting tail/probability of getting head (5 points).

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>qnorm(0.02)
[1] -2.053749
>qnorm(0.01)
[1] -2.326348
```

4. Let $X_1, \cdots, X_n$ be a random sample from Bernoulli distribution with parameter $p$.
   (a) Find the likelihood function and the maximum likelihood estimator of $p$ (5 points).
   (b) Find the maximum likelihood estimator of $p^2$ (5 points).
   (c) Find an unbiased estimator of $p^2$ (5 points).